AUTOMATING XFEM TECHNIQUE FOR CRACK ONSET OF COMPOSITES

Elruby, A. Y.[1] and Nakhla, S[2]*
[1] PhD Candidate, [2] Assistant Professor
Mechanical Engineering Department
Faculty of Engineering and Applied Science
Memorial University of Newfoundland, St. John's, NL, Canada
* Corresponding author (satnakhla@mun.ca)

***Keywords**: Failure, FEA, Prediction*

ABSTRACT

The eXtended Finite Element Method (XFEM) is one of the most versatile methods for solving crack propagation problems [1]. XFEM works by enriching the critical region(s) with special shape functions to account for crack propagation [2]. A noteworthy contribution for XFEM applications was done by its coupling with Level Sets Method (LSM) making it possible to predict the crack location and propagation direction [3, 4]. This method is currently implemented in finite element commercial code ABAQUS. Meanwhile, XFEM predictions for crack onset and propagation rely on the stress field of finite elements simulations. It is well known that stress fields tend to converge at a slower rate than that of displacements, making it difficult to accurately capture the crack behavior. Furthermore, identifying the critical region(s) rely mainly on skills of an expert user. In the presented work, a new approach is developed to automate identification process of potential crack onset region(s), eliminating the need for an expert user and minimizing the problem complexity. Hence, it allows non-expert users to precisely model crack problems in ABAQUS. Also it results in enriching critical region(s) only instead of enriching the entire model enhancing cost effectiveness. Moreover, the new approach is capable of selecting the optimized mesh size for simulations. Both features have a significant effect on computational efficiency and accuracy of predicted results. For this purpose a python script is developed into ABAQUS scripting interface implementing an iterative algorithm based on material-specific failure criterion. The developed technique is to capture the behavior of cracks in brittle materials such as matrix resins of composite materials. Hence, a brittle material failure criterion was used for crack onset. For the purpose of initial validation of the developed algorithm, a set of six concrete specimens are tested under four point bending loading. The predicted critical loads corresponding to crack onset showed an excellent agreement with measurements.

1 INTRODUCTION

Geometric discontinuities such as a sharp change in geometry, opening, hole, notch or a crack are known to be the main source of failure [5]. Discontinuities generate significant stress concentrations reducing the overall strength of the material [5]. For a crack problem, the stress field at the tip is singular and the conventional FEM can hardly capture the crack behavior. For instance, modeling crack onset and propagation problems in solid mechanics is quite problematic. Which is considered to be a non-smooth solution that can be dealt with based on one of two fundamental approaches [6]. The first is based on the classical FEM which requires mesh alignment with the crack at each step. As well as, refining the mesh near crack tip in order to capture the stresses accurately. This is not only computationally inefficient but also may affect accuracy of the results. In addition, this might be a thought-provoking process requiring an expert user to accurately model the problem. The second approach is based on enriching the approximation polynomials with special shape functions that can capture the behavior of discontinuities or singularities called 'enriched methods' [6]. The main advantage of the second approach is that the solution is independent of the mesh. This approach is a quite useful improvement compared to the first one. XFEM belongs to the second category which was first introduced by Ted Belytschko et al. [7, 2] in 1999 based on the Partition of Unity Finite Element Method (PUFEM) by Babuska et al. [8, 9]. References [10, 6] provide a comprehensive review of XFEM method. The proposed technique in the current study is using the capabilities of XFEM method for crack propagation while optimizing its efficiency regarding crack onset. It is noteworthy to mention that usually crack problems rely on notched beams for testing. This is essentially done to initiate or trigger the crack. Few studies were conducted to test un-notched beams. An earlier study by Hamad et al. [11], presented a numerical model where the cracked zone was modelled using the fictitious crack approach while any other zone was considered linearly elastic [11]. Hamad et al. [11] tested un-notched beams in their study. Their approach was based on the analytical model for fictitious crack propagation in concrete beams by Ulfkjær et al. [12]. Hamad et al. assumed pre-existing crack in their model. The prediction results showed a relatively good agreement compared to testing with an upper bound of more than 10%.

2 PROBLEM FORMULATION

As mentioned earlier, XFEM predictions for crack onset rely on the stress field of Finite Element (FE) simulations. Stress fields are known to converge at a slower rate than those of displacements. This arises the need of optimizing the mesh quality to assure predictions accuracy. Moreover, XFEM requires an expert user to identify critical zone(s) otherwise the entire domain will be enriched with special shape functions which in turn will increase the computational cost drastically. The aim of the current work is to provide a robust technique that can allow a non-expert user to accurately model a problem for optimizing crack onset and propagation in a structure without assuming crack location a priori. The new technique is developed using python scripting capabilities in the commercial Finite Element Analysis (FEA) software ABAQUS. The proposed technique can be used to account for different materials. In the sense of making the modeling process easier and computationally more efficient while eliminating the need of an expert user.

3 XFEM FUNDAMENTALS

In order to illustrate the XFEM fundamentals, consider the FE model of a cracked body shown in Figure 1. In this mesh there exists regular nodes, heaviside nodes and crack-tip nodes. Regular nodes are elemental nodes in which their elements are not cut by the crack (set *I*). Regular nodes follow the conventional FE shape functions formulation. On the other hand, heaviside nodes are elemental nodes wherein the crack is passing through (set *J*). Heaviside nodes are enriched with special shape functions to account for the crack discontinuities. Finally, the crack-tip nodes are those nodes surrounding the crack-tip (set *K*). The crack-tip nodes are enriched with special shape functions that can capture the crack propagation behavior based on the Stress Intensity Factor (SIF) at the crack-tip.

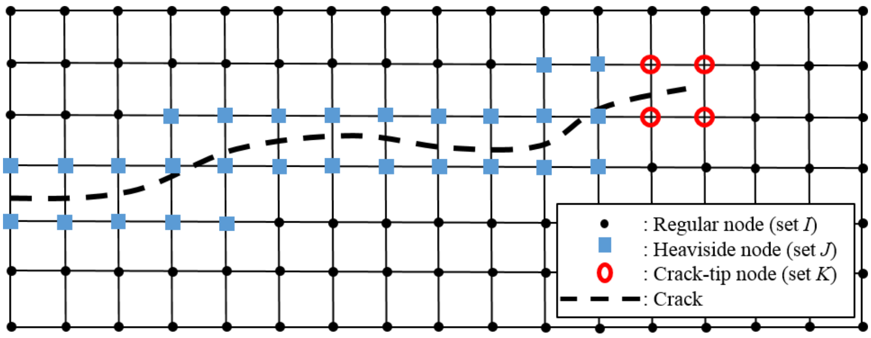

Figure 1. Finite element model of a cracked body

The special shape function for the XFEM method takes the form of Equation 1.

$$u^h(x) = \sum_{i \in I} u_i\, N_i + \sum_{j \in J} b_j\, N_j H(x) + \sum_{k \in K} N_k \left[\sum_{l=1}^{4} c_k^l F_l(x)\right] \tag{1}$$

Where x is the global displacement, N_i is the shape function of node i, u_i is the degree of freedom associated to node i, $H(x)$ is the heaviside function or jump function, N_j is the shape function interconnected to the jump function or discontinuity at node j, while b_j is the degree of freedom linked to the jump function (heaviside enrichment function), $F_l(x)$ is the crack-tip enrichment function, N_k is the shape function associated to the crack-tip function at node k and c_k^l are the additional degrees of freedom related to the elastic asymptotic crack-tip enrichment functions (presented by donut markers on Figure 1). Regarding the sets that describe the domain of each region, I is the set of all nodes in the entire mesh of the problem, J is the set of the heaviside enrichment nodes (those nodes whose shape functions are censored by the crack enrichments) and k is the one representing the set of crack-tip enriched nodes (those nodes whose shape functions are amended by the crack-tip enrichments). Throughout the domain of the problem, the regions which are neither enriched by a Heaviside function nor a crack-tip asymptotic function are solved using the regular conventional shape functions of a classical FEM problem. Hence, Equation 1 can be simplified to include only the first term on the Right Hand Side (RHS) leading to the conventional formulation of FEM as.

$$u^h(x) = \sum_{i \in I} u_i\, N_i \tag{2}$$

For the region which is cut by the crack (crack domain), the displacement approximation function of XFEM can be reduced to only include the first and the second terms of Equation 1. Which can be presented as follows.

$$u^h(x) = \sum_{i \in I} u_i\, N_i + \sum_{j \in J} b_j\, N_j H(x) \tag{3}$$

Finally, in order to account for the singularities at the crack-tip, Equation 1 can be reduced to only include the first and the third terms on the RHS. Which can be presented by.

$$u^h(x) = \sum_{i \in I} u_i\, N_i + \sum_{k \in K} N_k \left[\sum_{l=1}^{4} c_k^l F_l(x)\right] \tag{4}$$

For further details on the derivation of the method the reader is invited to consult with references [2, 3, 4, 7, 13].

4 METHODOLOGY

The material behavior is dominant for selecting the failure criterion. In the current work composite materials are the ones of interest. Most composite materials exhibit a brittle failure due to their low strain to failure capacity. Brittle failure is said to be catastrophic, once a crack is initiated it will propagate rapidly till fracture. Therefore accurate crack onset prediction for such material behavior is necessary. For that purpose, a maximum strain failure criterion is adopted for crack initiation. As for crack propagation, the evolution of the crack after being initiated is based on the material fracture energy. As mentioned earlier, the XFEM method is promising considering complex problems dealing with discontinuities and singularities. Some drawbacks of the method in its current phase are to be exemplified. First challenge is the dependency of predictions accuracy on mesh quality. Second challenge is related to the computational cost resulting from enriching the entire domain of the problem. The main aim of current work is automating the modeling process while maintaining desired accuracy for predicted failure results. For that purpose, an automation technique is proposed using Python scripting to perform three main tasks. First, the process of determining the optimum mesh size. An algorithm is implemented in the Python script for determining the optimum mesh size, which is strongly dependent on the model geometry. Second, the model is to be checked for failure or crack onset. The checking process is done by an iterative algorithm based on the selected failure criterion. Third, once critical region(s) are identified, the possible critical zone(s) for crack onset and propagation are enriched with special shape functions using the XFEM method for failure predictions.

5 FINITE ELEMENT ANALYSIS MODEL

A four point bending beam for concrete based on the ASTM C78 designation [14] is chosen to test and validate the proposed technique. Parameters for geometric dimensions, loading conditions, as well as the boundary conditions were set in the script based on the description of the aforementioned standard test. Figure 2 illustrates loading and boundary conditions of the four point bending beam problem. The total length of the beam is 400 mm centrally position on two supports spanned 300 mm apart. The beam is of square cross-section with a width of 100 mm and the load is applied using two concentrated forces on the mid segment of the supported span allowing the beam to experience uniform bending.

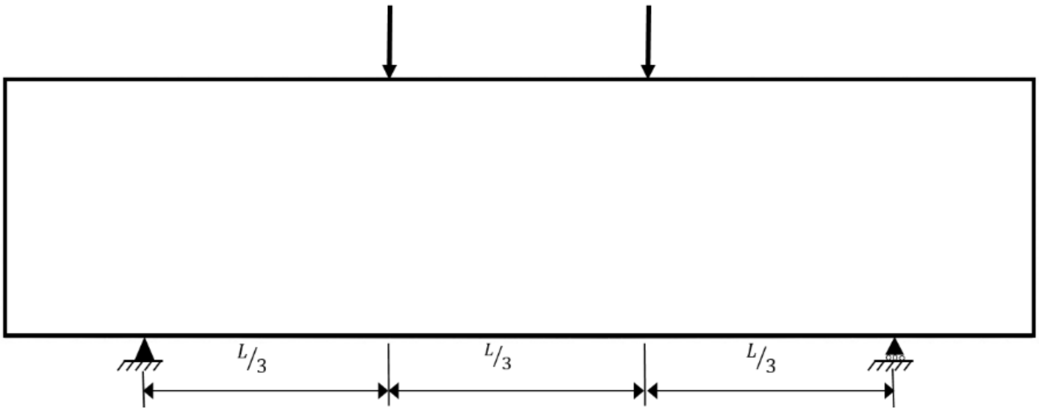

Figure 2. 2D Beam showing loads and boundary conditions

The material model for concrete is chosen to be linearly elastic in compression and tension until failure. For failure initiation, a traction separation law based on maximum principal stress is adopted taking into consideration the ease of determining the maximum tensile strength for concrete by testing. For the damage evolution, it was selected based on the fracture energy of concrete rather than the critical crack opening displacement. This is due to the difficulty of capturing a crack behavior after being initiated for a brittle material. Moreover, the research data available in literature on fracture energy of concrete are is quite more convenient. A general static step is chosen for the analysis. A 4-noded bilinear plane strain quadrilateral element (CPE4R) with reduced integration is selected for meshing. The model is meshed using structured meshing control in ABAQUS.

6 PROPOSED TECHNIQUE

The proposed technique has three main tasks to be performed. These tasks are going to be illustrated in this section. First, determining the optimum mesh size for the four point bending beam problem. A correlation between the optimum mesh size and the smallest dimension in the geometry is extracted. Which in turn lead to a factor relating the mesh size to the given geometry. This factor is used in the script to select mesh element size automatically. The convergence history of the normalized principal stress predictions is shown in Figure 3.

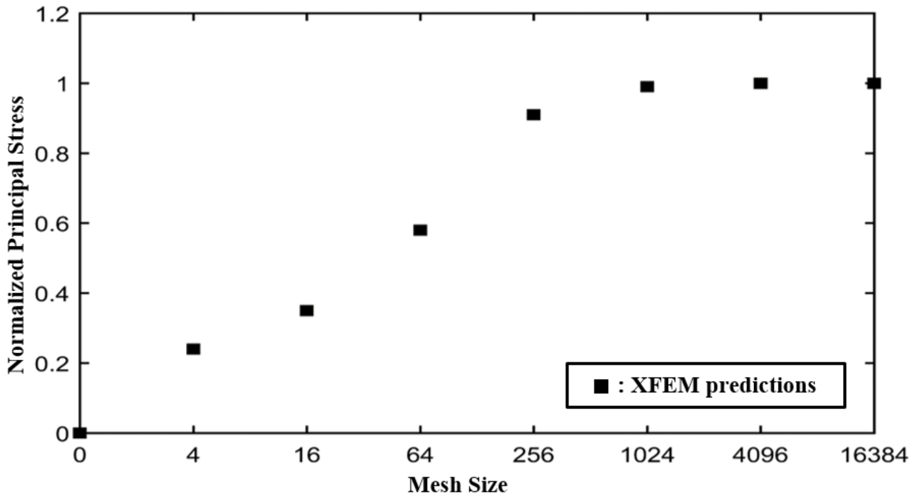

Figure 3. Normalized principal stress convergence

Second, the model is to be checked for failure or crack onset. The iterative algorithm dedicated for that purpose is shown in Figure 4. On each iteration the failure criterion is checked. In case the failure condition is not met, the load is incremented until the failure condition is encountered. During each iteration the script extracts data regarding the most probable region(s) identified for crack onset to occur. Extracted data are appended to a report with their associated strains and stresses.

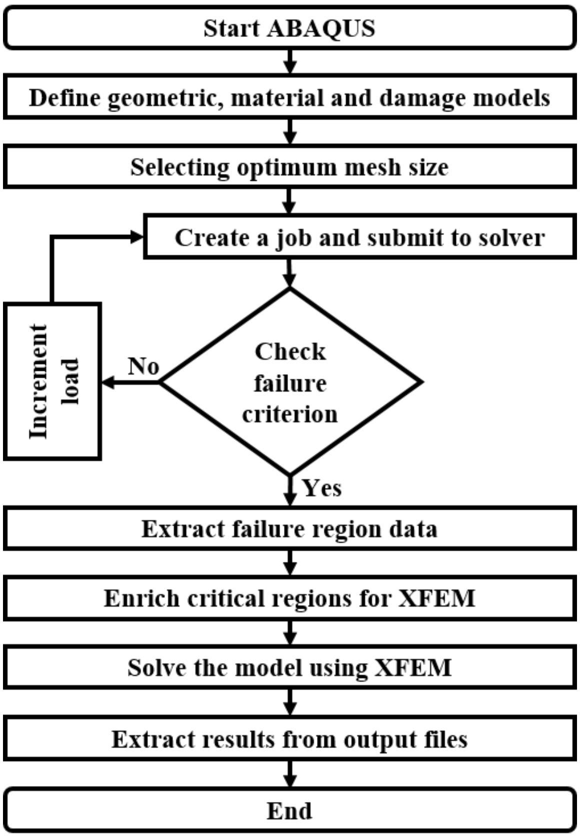

Figure 4. Flowchart of the proposed technique

Once the failure criterion is encountered, the script highlights the identified critical region(s) for crack onset as shown in Figure 5. This identified region is the one where the crack is most likely to initiate.

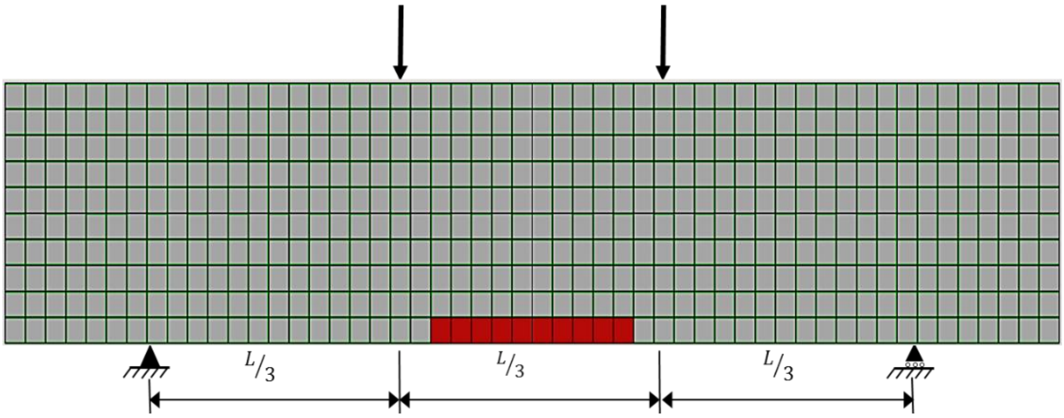

Figure 5. Critical region(s) for crack initiation highlighted on the FEA model

Third, the script automatically enriches possible critical zone(s) for propagation. The enriched zone(s) are shown in Figure 6. The enriched zone(s) for crack propagation are based on identified critical region(s) for crack onset. Once the model enrichment is accomplished, the model becomes ready for predicting crack onset and propagation.

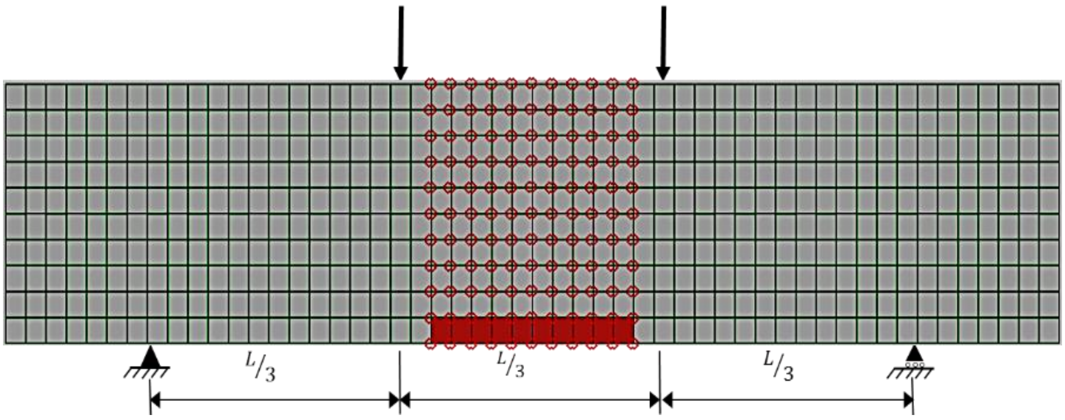

Figure 6. The FEA beam model showing the enriched zone(s)

Finally, the script submits a new job to the ABAQUS solver. This job is based on the XFEM enrichments. The use of enrichments capabilities enables simulating failure predictions regarding the crack onset and its propagation. Bearing in mind that critical region(s) have already been identified using the proposed technique. In other words, cracks are expected to initiate with in the identified region(s) only.

7 NUMERICAL PREDICTIONS, TESTING AND VALIDATION

The proposed technique identified the critical region(s) most likely to fail. In other words, the region(s) where the crack will initiate. For the four point bending problem, the critical region recognized by the proposed technique was shown on Figure 5. As can be seen from Figure 5, the script was able to highlight the area which experience the maximum bending moment in the tension side. Tensile strains in this region are more likely to grow, making this area the most critical one. Crack propagation zone(s) were automatically enriched by the script following the identification of the most critical region for crack onset. The enriched zone was shown in Figure 6. The proposed technique enriched only 25 % of the whole domain of the problem. This will have a significant effect on the computational cost for solving the same problem.

Six concrete specimens were tested at Memorial University of Newfoundland laboratories. The material mechanical properties for the specimens were measured from testing. First, the compressive strength was measured according to ASTM C39 standard testing [15]. Second, the flexural strength was measured according to ASTM C78 [14]. Third, splitting tensile strength was measured according to ASTM C496 [16]. Finally, the modulus of elasticity was measured according to ASTM C469 [17]. The measured data associated to the standard testing are recorded in Table 1. The Poisson's ratio was assumed to be 0.2.

Material	Compressive Strength (MPa)	Flexural Strength (MPa)	Splitting Tensile Strength (MPa)	Modulus of Elasticity (MPa)
Mix #1	60.08	8.40	3.43	32010
Mix #2	51.16	8.31	3.07	30910
Mix #3	45.90	5.40	3.20	29300
Mix #4	81.09	10.56	5.41	34060
Mix #5	74.07	9.66	3.91	35500
Mix #6	41.49	6.57	3.32	27470

Table 1. Mechanical properties from testing the six concrete specimens (MPa = 10^6 *N/m^2)

The fracture energy of concrete $\boldsymbol{G_f}$ was calculated using Equation 5 as introduced in [18].

$$G_f = G_{f_o}\left(\frac{f_c+8}{10}\right)^{0.7} \tag{5}$$

Where G_f represents the calculated fracture energy of plain concrete, G_{f_o} of 26 N/m is used in the current study to correlate with the aggregate size used in preparing the specimens. The fracture energy is dependent on two main parameters. The size of aggregate used and the compressive strength f_c of the mixture used. The fracture energy for each specimen was calculated and recorded in Table 2. The failure load measured from testing $F_{Testing}$ was recorded in Table 2 for each specimen. The load cell accuracy of the used testing machine is ±0.062 %.

Material	$\boldsymbol{G_f}$ (N/M)	$F_{Testing}$ (N)	$F_{Predicted}$ (N)	Error (%)
Mix #1	99.56	28000	28542	-1.94
Mix #2	90.24	27700	28238	-1.94
Mix #3	84.54	18000	18343	-1.91
Mix #4	120.19	35200	35883	-1.94
Mix #5	113.48	32200	32832	-1.96
Mix #6	79.64	21900	22326	-1.95

Table 2. Calculated fracture energy G_f, Testing failure load, Predicted failure load and Error

Finally, the predicted failure load $F_{Predicted}$ from the XFEM simulation results using the proposed technique is tabulated for each corresponding mixture. Predictions represented an upper bound when compared to testing results with an average approximate error of 2%. Figure 7 is depicted for showing the actual cracks from testing presented by solid lines compared to the predicted ones presented by dashed lines.

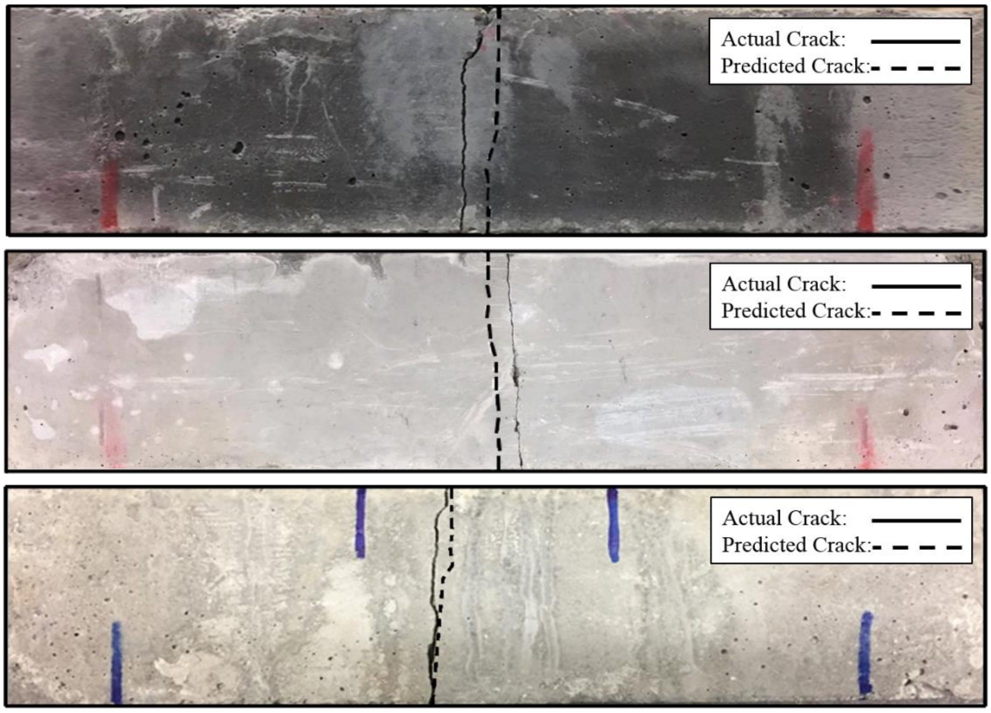

Figure 7. Cracked Specimens after testing compared to the proposed technique predictions

8 CONCLUSIONS AND FUTURE WORK

The proposed technique was able to identify the critical region(s) for the four point beam bending problem. Computational effort was reduced by more than 70%. The technique omitted the need of an expert user for modeling the current problem. In terms of failure load, an upper bound was observed comparing predictions to actual testing results with an average error of 2%. The predicted results of crack onset and propagation are in an excellent agreement with testing as shown in Figure 7. Based on the current study, the proposed technique is very promising. First, automating the process of failure prediction might be useful for similar studies. Second, in terms of computational efficiency the technique showed a strong reduction in computational efforts. Third, regarding the failure load based on the problem in hand, the predictions showed an outstanding agreement with the testing results. Finally, the crack onset location as well as its propagation predicted by the technique are in an excellent agreement with the testing results. Currently, Lamination resin materials are being tested. For future work, the current technique is to be extended for testing more complex material behavior. Reinforced composites are to be examined using the proposed technique. Three dimensional problems are to be considered in the future, as well as contact problems. . Afterwards, laminated composites are to be studied using the proposed technique.

REFERENCES

[1] Z. Zhuang, . Z. Liu, B. Cheng and J. Liao, "Extended Finite Element Method", Elsevier and Tsinghua University Press computational mechanics series, 2014.

[2] J. Dolbow, "An extended finite element method with discontinuous enrichment for applied mechanics," Northwestern university, 1999.

[3] M. Stolarska, D. Chopp, N. Moes and T. Belyschko, "Modelling crack growth by level sets in the extended finite element method," *International Journal of Numerical Methods,* pp. 943-960, 2001.

[4] A. Gravouil, N. MOËS and T. Belytschko, "Non-planar 3D crack growth by the extended finite element and level sets—Part II: Level set update," *International Journal for Numerical Methods in Engineering,* vol. 53, no. 11, pp. 2569-2586, 2002.

[5] S. Mohammadi, XFEM fracture analysis of composites, John Wiley & Sons, 2012.

[6] T.-P. Fries and T. Belytschko, "The extended/generalized finite element method: An overview of the method and its applications," *International Journal for Numerical Methods in Engineering,* vol. 84, no. 3, pp. 253-304, 2010.

[7] T. Belytschko and T. Black, "Elastic crack growth in finite elements with minimal remeshing," *International Journal for Numerical Methods in Engineering,* pp. 601-620, 1999.

[8] Babuska, Andersson, Guo, Oh, Melenk and Maryland, "Finite Element Method for Solving Problems with Singular Solutions," *Univ College Park Inst FOR Physical Science Technology,* 1995.

[9] I. Babuška and J. M. Melenk, "The Partition of Unity Method," *International Journal of Numerical Methods in Engineering,* vol. 40, no. 4, pp. 727-758, 1997.

[10] A. Yazid, N. Abdelkader and H. Abdelmadjid, "A state-of-the-art review of the X-FEM for computational fracture mechanics," *Applied Mathematical Modelling,* vol. 33, no. 12, pp. 4269-4282, 2009.

[11] W. I. Hamad, J. S. Owen and M. F. M. Hussein, "An efficient approach of modelling the flexural cracking behaviour of Concrete prisms subject to monotonic and cyclic loading," *Engineering Structures,* no. 51, pp. 36-50, 2013.

[12] J. P. Ulfkjær, S. Krenk and R. Brincker, "Analytical model for fictitious crack propagation in concrete beams. , 121(1), 7-15.," *Journal of Engineering Mechanics,* vol. 121, no. 1, pp. 7-15, 1995.

[13] N. Moes, J. Dolbow and T. Belytschko, "A finite element method for crack growth without remeshing," *International Journal for Numerical Methods in Engineering,* vol. 46, pp. 131-150, 1999.

[14] C. ASTM, *"Standard Test Method for Flexural Strength of Concrete (Using Simple Beam with Third-Point Loading).",* Philadelphia, PA: American Society for Testing and Materials, 1999.

[15] C. ASTM, *"Standard test method for compressive strength of cylindrical concrete specimens.",* Philadelphia, PA: American Society for Testing Materials, 1996.

[16] C. ASTM, *"Standard Test Method for Splitting Tensile Strength of Cylindrical Concrete Specimens.",* philadelphia, PA: American Society for Testing and Materials, 2003.

[17] C. ASTM, *Standard test method for static modulus of elasticity and Poisson's ratio of concrete in compression.",* Philadelphia, PA: American Society for Testing and Materials, 2002.

[18] Z. P. Bažant, ""Analysis of work-of-fracture method for measuring fracture energy of concrete."," *Journal of Engineering Mechanics,* vol. 2, no. 122, pp. 138-144, 1996.