A PRACTICAL ANALYTICAL MODEL FOR PREDICTING RESPONSE OF 3D FIBERGLASS FABRIC SUBJECTED TO IMPACT LOADING

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ABSTRACT

The main objective of the present study is to develop a practical analytical model for use by practicing engineer for establishing the response of a new class of 3D fiberglass fabrics (3DFGF) subject to low-velocity impact loading. 3DFGF is a relatively recently developed fabric which consists of two layers of woven fabrics knitted together by a series of pillars. The vertical pillars create a core cavity which may be filled with any suitable foam to enhance the mechanical properties and energy absorption capacity of the fabric under impact loading. The superior mechanical performance of the fabric was established and demonstrated in our earlier studies, thus rendering it as a competitive material of choice for applications in automotive. However, due to the complex structure of 3DFGF, its mechanical response cannot be predicted by the available methodologies. Consequently, it is essential to develop a model by which the response of structures made by this material could be predicted, thus facilitating its use in industrial applications. In one of earlier works, a finite element (FE) model was developed to simulate the mechanical response of 3DFGF under various loading conditions. The FE results were verified against experimental results. However, acquiring the response of this material with its complex configuration under complex loading conditions (e.g., impact loading), by FE simulation requires advanced skills, involving significant time and effort. Therefore, it is prudent to develop an analytical model by which the task could be conducted by most practicing engineers.

The proposed analytical model is developed based on the energy balance principle. It is assumed that the impact energy is absorbed mainly by two mechanisms (i.e., bending, and contact), while shear and membrane actions also play roles. However, it is assumed that the shear and membrane effects are negligible in comparison to the bending and contact effects. In order to consider the bending effect, the bending energy of a 3DFGF panel subjected to a point load is considered. Moreover, the modified Hertz contact law is used to account for the contact mechanism. The contact parameters are calculated by conducting FE simulation of an indentation experiment on a 3DFGF panel. Subsequently, the analytical equations are developed to predict the maximum sustained impact load, maximum deformation and contact time. The analytical results are verified by comparison against FE and experimental results. The results indicate that the analytical model can reliably predict the impact response of the panels prior to development of a damage or crack on them, which would be the governing criterion in practical designs.

1 INTRODUCTION

3D fiberglass fabric (3DFGF) is a new generation of truly 3D fabrics, which was recently introduced to the market. The main applications of 3DFGF are in marine, wind turbines, vessels and infrastructures. The most significant characteristics of this fabric are: high skin-core deboning resistance, impact resistance, light weight and high specific stiffness and strength. These attributes render this fabric as a great candidate for the applications and more. Recently, the authors developed a new class of fiber metal laminates, incorporating the 3DFGF, with the main objective of its application in transport vehicles [1-4]. Since 3DFGF is a newly developed material, there are a limited number of research works that have considered their mechanical properties and performance. Therefore, to gain a better understanding of the mechanical behavior of this fabric, its response under various loading conditions was evaluated by the authors. An earlier study [5] investigated the mechanical behavior of 3DFGF under bending, flatwise compression and low-velocity impact loading conditions. Two different thicknesses of 3DFGF (i.e., 4 mm and 10 mm) were considered to investigate the influence of thickness on the performance of 3DFGF. The inherent structure of 3DFGF provides hollow cores, which could be filled with a foam to further enhance its mechanical properties. Figure 1 shows the 3DFGF with its cores empty and filled with a foam, respectively. The mechanical performance of these panels was compared against one-another. In addition, their response under low-velocity impact generated by different sizes of impactors was also investigated [6]. The most interesting attribute of the response under impact was the absence of any delamination emanating from the impact, which is quite significant from structural mechanics' perspective. Overall, this fabric offers superior energy absorption capacity in comparison to its conventional laminated composite and FML counterparts.

The main aim of the present study is to develop an analytical model for predicting the response of 3DFGF under low-velocity impact loading. The energy balance principle is used to develop the model. It is assumed that the impact energy is absorbed by two mechanisms (i.e., bending and contact). The modified Hertz contact law is considered to account for the effect of contact. The required contact parameters are calculated through simulating the response of the panel subjected to an indentation test using the commercial FE software, ABAQUS. Three analytical equations are developed to predict the maximum impact load, maximum deformation and contact duration of 3DFGF panels subjected to impact loading. The analytical results are compared against FE and experimental results to validate the integrity of the proposed model. Moreover, in order to effectively understand the impact behavior of 3DFGF, the contributions of bending and contact mechanisms in absorbing the impact energy are calculated and compared.

(a) (b)

Figure 1. The 3D fiberglass fabric (3DFGF) with its cores (a) without foam, (b) with foam.

2 DEVELOPMENT OF THE ANALYTICAL MODEL

2.1 Maximum Sustained Impact Load

As stated, the impact response of 3DFGF is analyzed based on the energy balance model, developed based on the energy conservation principle. It is assumed that at the maximum deformed state, the entire kinetic energy of the impactor is transferred to the target, causing it to deform in a quasi-static mode. The energy balance model can be presented as follows:

$$\frac{1}{2}mv^2 = E_c + E_b + E_s + E_m \quad (1)$$

where E_b, E_s and E_m refer to the energy absorbed through bending and shear deformation of the panel, and membrane stretching of its skins, respectively. E_c is the amount of absorbed energy as a result of the friction contact created by the indentation mechanism. The contact effect can be calculated using the generalized form of Hertz contact law [7]:

$$P = C\alpha^n \tag{2}$$

where C and n are the contact stiffness parameters, and P and α are the applied force and associated indentation, respectively. In the case of purely elastic response, n is taken as 1.5. However, in the case of sandwich structures, the relatively lower modulus of the core in turn reduces the value of n. Therefore, the absorbed energy due to contact, E_c, can be calculated by [8]:

$$E_c = \int_0^{\alpha_{max}} P d\alpha \tag{3}$$

Substitution of (1) into (2) and its integration yields

$$E_c = \frac{(P_{max})^{\frac{n+1}{n}}}{C^n(n+1)} \tag{4}$$

It should be noted that the resulting membrane and shear effects are much smaller in magnitude in comparison to the indentation and bending deformation magnitudes. Therefore, the force-displacement relationship for a fixed supported circular plate subjected to a point load at its center can be represented by

$$P = K_b\Delta \tag{5}$$

where Δ is the overall deflection of the plate without consideration of any indentation, and K_b is the bending stiffness of the plate, which can be evaluated by [9]

$$K_b = \frac{4\pi E_r h^3}{3(3+\nu_r)(1-\nu_r)a^2} \tag{6}$$

with

$$E_r = E_c r^3 + E_l(1-r^3) \tag{7}$$

and

$$\nu_r = \nu_c r + \nu_l(1-r), \quad \text{where: } r = \frac{c}{h} \tag{8}$$

In above equations, E_l, ν_l, E_c and ν_c are the Young's modulus and Poisson ratio of the skin and core, respectively. C, h and a are core thickness, total thickness of the panel and radius of the panel, respectively. Therefore, the absorbed energy due to bending can be calculated by

$$E_b = \frac{P_{max}^2}{2K_b} \tag{9}$$

As a result, the energy balance represented in (1) can be rewritten as

$$\frac{1}{2}mv^2 = \frac{\left[{P_{max}}^{n+1}/C\right]^{1/n}}{(n+1)} + \frac{3(3+\nu_r)(1-\nu_r)a^2P_{max}^2}{8\pi E_r h^3} \tag{10}$$

The values of C and n can be obtained through an indentation test conducted on a panel. Knowing these two parameters and bending stiffness, one can calculate the value of P_{max} for a given impact energy.

2.2 Maximum Deformation and Contact Time

Mass and velocity of the impactor and target are considered as m_1, v_1, m_2 and v_2, respectively. When the impactor and target come in contact, the rates of change of impactor's velocity during impact can be presented by [10]

$$m_1\frac{dv_1}{dt} = -P \text{ and } m_2\frac{dv_2}{dt} = -P \tag{11}$$

$$\dot{\alpha} = v_1 + v_2 \tag{12}$$

In above, α is the indentation due to impact force, which is equivalent to the distance by which the impactor and target approach one another. Differentiation of (12) and substitution of (11) and (2) into the resulting equation gives

$$\ddot{\alpha} = -CM\alpha^n \qquad \text{where: } M = \frac{1}{m_1} + \frac{1}{m_2} \tag{13}$$

By multiplying both sides of (13) by $\dot{\alpha}$ and integrating, one obtains:

$$(\dot{\alpha}^2 - v^2) = -\frac{CM}{n+1}\alpha^{n+1} \tag{14}$$

where v is the velocity of the impactor and target at the onset of contact. It should be noted that α_{max} occurs when $\dot{\alpha} = 0$, then

$$\alpha_{max} = \left[\frac{(n+1)v^2}{CM}\right]^{1/n+1} \tag{15}$$

In order to calculate the contact duration, it is assumed that the maximum deformation occurs at $0.5t_0$ where t_0 is the impact duration. First, equation (14) is rewritten in the following form

$$\dot{\alpha} = \sqrt{v^2 - \frac{CM}{n+1}\alpha^{n+1}} \tag{16}$$

Then, $\dot{\alpha} = \frac{d\alpha}{dt}$ is substituted in (16), yielding the value of dt as:

$$dt = \frac{d\alpha}{\sqrt{v^2 - \frac{CM}{n+1}\alpha^{n+1}}} \tag{17}$$

Combining (15) and (17), and letting $\delta = \alpha/\alpha_{max}$, following the integration of the resulting expression leads to

$$dt = \frac{\alpha_{max}}{v}\frac{d\delta}{\sqrt{1-\delta^{n+1}}} \tag{18}$$

So, it is assumed that the total impact duration is twice of the time that maximum deformation occurs. The total impact duration, t_0 can now be calculated by integration of (18) between the boundaries of [0,1]; here, the 0 represents start of the event, while 1 represents end of the event.

$$t_0 = \frac{2\alpha_{max}}{v}\int_0^1 \frac{d\delta}{\sqrt{1-\delta^{n+1}}} \tag{19}$$

3 Finite Element Simulation

The structure of 3DFGF is quite complex, and must be judiciously simplified for one to conduct a reasonably economical computational simulation. As explained earlier, 3DFGF is comprised of two layers of woven fiberglass fabric, knitted together by a series of fiberglass pillars. The core cavities of fabric are filled with a polyurethane foam to further improve the overall mechanical response of the fabric. The two layers of fiberglass fabric are simulated as an orthotropic material, with the use of Hashin damage criterion for establishing the failure mechanism. The mesh is generated using the reduced integration shell element (S4R) available in ABAQUS/Explicit.

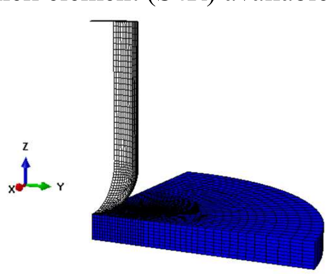


Figure 2. Quarter symmetry model of the 3DFML and impactor

Materials	Mechanical Properties
Fiberglass	ρ = 1750 kg/m^3, E_1 = 37 GPa, E_2 = 8 GPa, v_{12} = 0.31, G_{12} = G_{13} = 4.7 GPa, G_{23} = 3.2 GPa, X^T = 780 MPa, X^C = 750 MPa, Y^T = 50 MPa, Y^C = 120 MPa
Combined foam/pillars	ρ = 128 kg/m^3, E = 50 MPa, v = 0, K = 1.1, K_t = 0.8

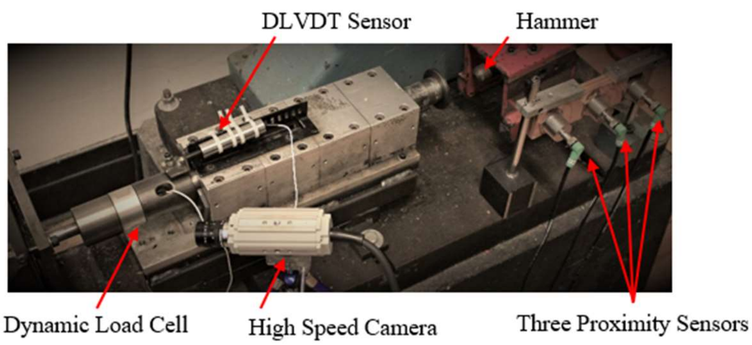
Table 1 – Mechanical Properties of Materials

4 Experimental Investigation

The 3D fiberglass fabric used in this study was acquired from China Beihai Fiberglass, Co. Ltd. (Jiujiang City, China). The core cavity of 3DFGF is filled with the polyurethane foam to improve its mechanical properties. The polyurethane foam used in this study was a two-part foam, supplied by US Composites (West Palm Beach, FL 33407). Furthermore, the resin used for this study was Araldite LY 564 (Bisphenole-A), low viscosity and warm curing epoxy Resin, along with Aradur 2954 (cycloaliphatic polyamine) hardener, supplied by the Huntsman Co. (West Point, GA).

The manufacturing of 3DFGF was a two-step process. First, the resin was applied on 3D fabric and then cured for two and eight hours at 60 ˚C and 120 ˚C, respectively. Afterwards, the two parts of polyurethane foam were mixed and injected into the core cavities of 3DFGF.

The low-velocity impact tests were carried out using a modified Charpy impact test equipment. The test system is equipped with a dynamic load cell, a dynamic linear variable differential transformer (DLVDT), and three proximeter sensors. The latter sensors are used to trigger the data acquisition system that enables a high-speed camera to record the event and capture the data related to the impact load, deformation and velocity. The Charpy pendulum hits the tup, which is guided through a series of bearing, ensuing a concentric loading application. Figure 3 depicts the impact equipment, the installed sensors and the fixture holding a specimen.



(a)

(b)

Figure 3. (a) the Impact test equipment, (b) the fixture used to hold the specimen

5 Results and Discussion

5.1 Contact Effect

The required contact parameters of the proposed model were established by conducting FE simulation of the indentation experiment that was carried out, by which a flat 3DFGF panel was loaded by a hemispherical indenter, and the resulting displacement was measured. The FE model was calibrated to model the load versus indentation curve captured through the experiment, thereby establishing the values of parameters C and n used to describe the contact law (see equation (2)). The contact load-indentation curve for 3DFGF is presented in Figure 4, in which the calculated C and n are also noted. As mentioned earlier, the value of n for sandwich panels is less than 1.5 due to the lower value of core modulus. Therefore, equation (10), which represents the relation between the impact energy and impact load, can be rewritten based on the contact parameters, thus facilitating the direct solution of P_{max}.

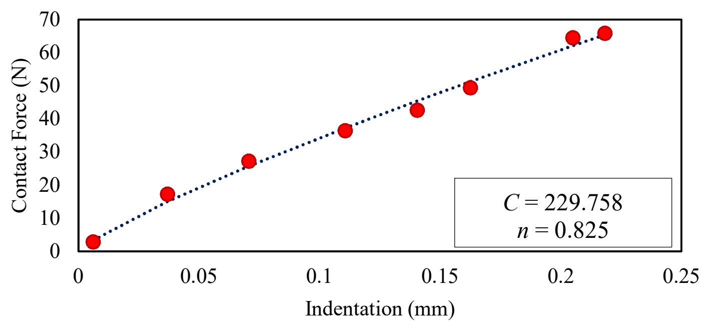


Figure 4. Contact force-indentation response of the 3DFGF developed by a hemispherical indenter

5.2 Impact Response

The maximum impact force for a given impact energy can be calculated using equation (10). Figure 5 depicts the variation of maximum sustained impact load as a function of applied impact energy. The predicted analytical impact resistance loads are compared against those calculated using the FE model. The results imply that, under the lower impact energies, the analytical model could reliably predict the maximum impact load sustained by the panel. However, once a damage (i.e., a crack) is developed on the target, the deviation between the predicted values and FE results expectedly increases, since the model does not account for any damage initiation. Notwithstanding, the maximum difference between the FE and analytical values is less than 30%. This would indicate that in real practice, where safety factors of two or higher are often used, the analytical model could be effectively used to predict the response of such complexly configured panels in a relatively reliable manner, even when the panel has succumbed to damage.

Moreover, equation (15) is used to calculate the maximum deformation of the 3DFGF experienced as a result of the impact. Figure 6 illustrates variation in the maximum deformation as a function of the applied impact energy. As seen, the maximum deformation of target increases as the applied energy is increased. Similarly, a good agreement is seen between the results predicted by the model and those produced by the FE simulation prior to development of damage on the target.

The contact duration of impact events was calculated using equation (19) and compared against the FE results. The results depicted in Figure 7 shows an ascending trend in the analytical results. However, the trend for contact time calculated by FEM has a descending trend. Similar to the model's predictions of the maximum impact load and deformation, the difference between the predicted and FE results is small under relatively low-impact energies, but it increases at the higher impact energies, which cause damage to the material. As seen, the reliability of model in predicting the maximum impact load and deformation is greater than when predicting the contact time.

As explained earlier, the analytical model is developed based on the energy balance methodology; accordingly, it has been assumed that the impact energy is absorbed by bending response of the panel, and the friction contact created between the impactor and the panel. To gain a better perspective of the response of 3DFGF when subjected to impact loading, the amount of impact energies absorbed through bending and contact are calculated and compared in Figure 8. The results indicate that under relatively low impact energies, the energy is absorbed by bending and contact in equal proportion. However, as the impact energy increases, the effect of bending becomes slightly greater than that by contact. It can be seen that essentially, once damage is developed in the target, the percentage of energy absorbed by bending and contact are shifted to approximately 70% and 30%, respectively.

To further examine the integrity of both the analytical and FE results, the results were compared against experimental results. In the experiment, a series of 3DFGF panels were subjected to five different levels of impact energy. The maximum sustained impact load and deformation of impacted specimens are compared against the numerical and FE results as presented in Figure 9. The error margin (% error) between the FE and analytical results with respect to the experimental results are calculate and reported in Table 2. As can be seen, both FE and analytical models can perform accurately before a damage is developed in the target, after which the margin of error is increased, though in a much lower rate in the case of FE results.

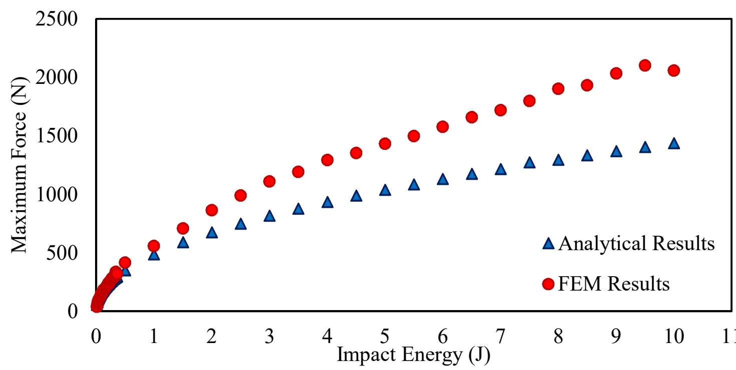


Figure 5. Variation of the maximum sustained impact load as a function of applied impact energy

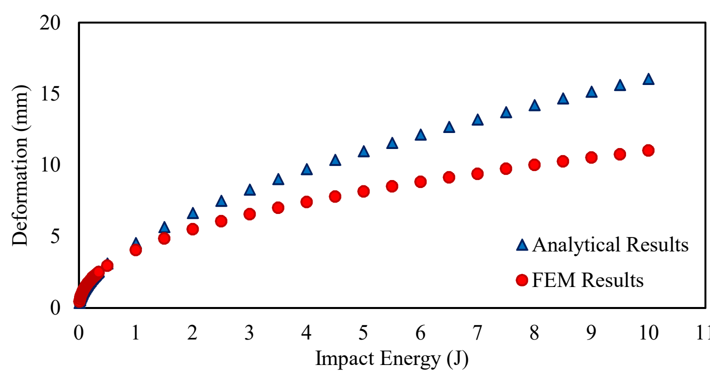


Figure 6. Variation of the maximum deformation as a function of applied impact energy

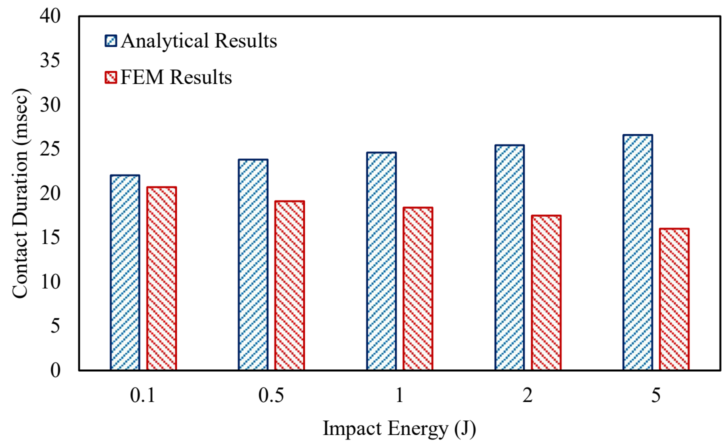


Figure 7. Variation of the contact duration as a function of applied impact energy

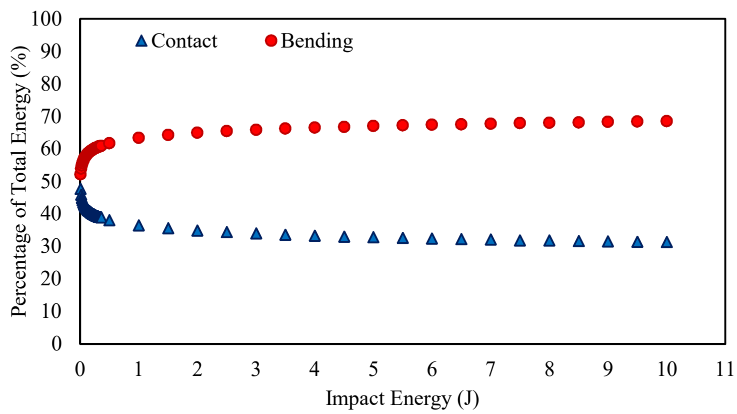
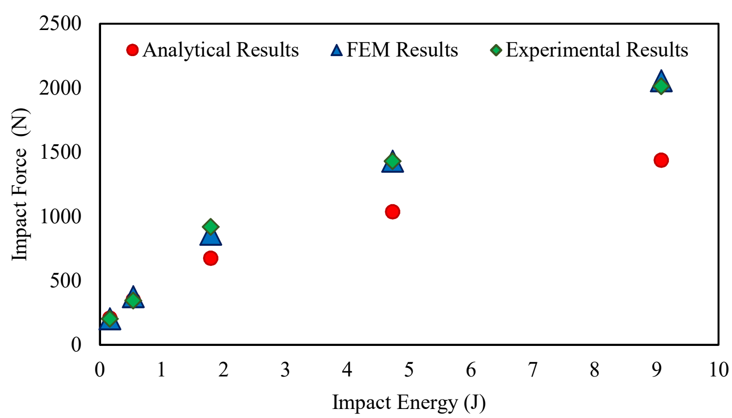
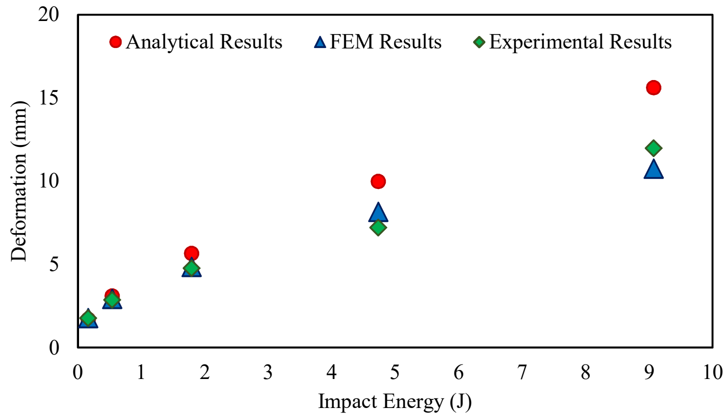


Figure 8. Percentages of absorbed energy due to contact and bending for different impact energy



(a)



(b)

Figure 9. Comparison of the results predicted by the analytical and FE models with the experimental values (a) variation of the sustained impact force, and (b) deformation, as a function of applied impact energy

Impact Energy	FE Results		Analytical Results	
(J)	Impact Load	Deformation	Impact Load	Deformation
0.15	0.39%	0.22%	2.56%	3.93%
0.50	8.28%	1.90%	0.93%	7.52%
1.80	6.40%	1.49%	26.70%	18.64%
4.70	0.03%	12.89%	27.42%	32.02%
9.10	2.10%	10.30%	28.67%	30.21%

Table 2. Error margin between the results produced by the analytical and FE models with respect to the experimental results

6 Conclusion

The main goal of present study was to develop an analytical solution by which the impact response of a new class of recently developed 3D fiberglass fabric could be predicted, thereby facilitating the use of the fabric by practicing engineers in various designs. A methodology, based on the energy balance principle, was used to develop the model. It is assumed that the impact energy is absorbed by bending reaction of the target and through the contact created between the impactor and target. The effect of bending energy was calculated based on the deformation of the panel loaded at its center. To establish the energy absorbed through contact, the generalized Hertz contact model was considered, which could be described using two contact parameters. The values of the parameters were established by performing indentation simulation of the target using commercial FE software ABAQUS. The proposed analytical model was used to calculate the maximum value of sustained impact load and deformation, as well as the contact time. The integrity of the proposed model was examined by comparing its results to the results obtained by FE simulation and experimental results. The comparison revealed that the results predicted by the analytical model are reliable and reasonably accurate in modeling the event up to the stage that a damage is initiated on the target, after which the accuracy of the results is suffered after this stage. Given the fact that in practice structural panels are designed to sustain load without developing a damage, one can conclude that the proposed model can be rendered as a practical design tool by practicing engineers.

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8 References

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