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ABSTRACT

The wrinkling of sandwich structure is a significant failure mode and should be considered in the design stage. Lack of accurate prediction of wrinkling may cause catastrophic failure as it can trigger other failures such as core failure and the debonding of face-sheets and core. Finite element analysis is employed using ABAQUS to investigate the wrinkling behavior. Finite element models are constructed based on the geometric and materials properties of test specimens available in the literature. An investigation on the effect of boundary conditions on the wrinkling is conducted. Also, the effect of element type used to model the face sheet is studied delineating the advantages and disadvantages of different elements. Classical analytical solutions of wrinkling are compared to finite element and test predictions in order to identify their restrictions.

1 INTRODUCTION

Sandwich structures are of great significance in engineering due to the high stiffness to weight ratio, excellent corrosion resistance and tailorable mechanical properties [1]. Typical sandwich structures comprise two face-sheets at sides and one core in the middle. The face-sheets are bonded to the core, which plays the role of providing support and stability to the face-sheets. Common failure modes of sandwich structures include face-sheet due to normal stress, core failure due to shear, face-sheet indentation due to impact or concentrated loads, and face-sheet global and local buckling. Local buckling is referred to as wrinkling and characterized by short wavelength buckling in the same order of magnitude as core thickness [2]. Wrinkling failure is critical because it may trigger other failure modes. Single-sided wrinkling, symmetric wrinkling and anti-symmetric wrinkling, Figure 1, are the three common wrinkling modes. The first mode usually occurs when sandwich structures are subjected to bending loads or when one face-sheet is much stiffer than the other. While symmetric and anti-symmetric wrinkling occur in the case of in-plane compression loading. The occurrence of symmetric or antisymmetric wrinkling mode is related to the material and geometric properties of the sandwich structures. Wrinkling failure is under continuous investigation since the 1940s analytically, experimentally and numerically. Several analytical solutions to predict wrinkling loads are available in literature. These analytical solutions are often compared to testing and considered classical approaches for design. However, it is not easy to conduct testing ideally due to the constraints of geometric imperfections caused by the specimen manufacturing process and the limitations from apparatus. Besides, complex failure modes and their interactions make it difficult to accurately determine the critical wrinkling load. Hence, finite element analysis (FEA) is often used to simulation specific test conditions. The use of FEA enables the user to account for manufacturing and geometric imperfections and realistic load application and boundary conditions. In

this pape finite element analysis are applied to investigate wrinkling failure mainly to identify the effect of boundary conditions, element selection and imperfections on predictions.



Figure. Different wrinkling modes, *a*) single-sided, *b*) symmetric and *c*) anti-symmetric

The wrinkling of sandwich structure was investigated in [3]. In the paper, eight different combina- tions of facesheets and supporting medium were studies analytically. Hoff and Mautner [4] presented buckling formulas for both symmetric and anti-symmetric modes based on the principle of minimum total potential energy. The symmetric wrinkling mode was also solved using the elasticity theory in their paper. Nardo [5] derived the exact solutions for the buckling of sandwich panel with loaded edges clamped and unloaded edges simply supported. Norris et al [6] summarized classical analytical research comparing with experiments. A brief overview of classical solutions is introduced in this article. In Plantema's model [7], an exponential decay of the core displacements in the transverse direction was assumed. The principle of minimum total potential energy was also applied in his model. Allen's model [8] had the same geometric assumptions as that of Plantema. The critical stress was derived by solving the governing equation with the assumption that the core stress field satisfied Airy's stress function. Analytical, numerical and experimental investigations of face-sheet wrinkling subjected to uniaxial and biaxial compression in sandwich shell was conducted by Stiftinger and Rammerstorfer [9]. The effect of the orthotropy of face-sheet and core were investigated and it was found that anti-symmetric wrinkling was critical for face-sheets with isotropic core, and symmetric wrinkling was possible for face-sheets with the orthotropic core. Niu and Talreja [10] presented a unified model to incorporate symmetric wrinkling and anti-symmetric wrinkling. The Winkler's model was modified according to their model and the two parameter model was evaluated in their paper. The Winkler's elastic foundation approach can also be used in predicting the symmetric wrinkling load of sandwich structure, as explained in [2]. The core was modeled as elastic spring and the critical stress was derived by solving the differential governing equation.

Vonach and Rammerstorfer [11] presented an analytical model for the wrinkling of thick orthotropic sandwich plates bonded to a transversely isotropic thick core. A 3D finite element analysis simulation using ABAQUS was used to validate the analytical model. Hadi [12] used finite element analysis to study sandwich column wrinkling and compared FEA results with the existing analytical and experimental results in [13-15]. In the FEA model, the face-sheet was modeled using four noded quadrilateral elements based on the Mindlin-Reisser shell formulation. Fagerberg and Zenkert [16] investigated the reason of discrepancies between experimental and analytical results. They presented an imperfection induced wrinkling model, which includes geometric imperfections of sinusoidal shape. The accuracy of the analytical model was also compared using FEA simulation. Plane stress 2D finite element analysis simulation was conducted in ABAQUS. Periodic boundary conditions were implemented similar to [11]. Ji and Waas [17] investigated the global and local buckling of a sandwich beam using classical elasticity. Finite element analysis using Abaqus was applied to validate the accuracy of their model. Fleck and Sridhar [18] conducted experiments to investigate the failure mode of sand- wich columns comprising of GFRP face-sheet and PVC foam core under the end compression. For face-sheet wrinkling, modified Hoff's model [4] was used as the analytical solution in which the leading coefficient is reduced to be 0.5.

2 FINITE ELEMENT MODELLING

In this section, two-dimensional finite element analysis is employed using ABAQUS to simulate the wrinkling of sandwich structures under in-plane compressions. The material properties and specimen geometry are extracted from existing test documented in literature. FEA results are compared with that of analytical models and test results in literature.

2.1 Description of the finite element model

In two-dimensional finite element model, the sandwich structure is divided into two regions corresponding to facesheets and core. Four-node bilinear plane stress solid elements are used for the core. The face-sheet is modeled by either quadratic beam element B23 or four-node bilinear plane stress element CPS4. The advantage of using beam elements is that the model is simplified hence the computational time is reduced, while the drawback is that the beam elements are assigned to the nodes in the top and bottom surfaces of the core to connect them to the core solid elements. Consequently, the distance between the centerline of the face-sheet and the interface of core and facesheets is neglected. Considering the much smaller face-sheets thickness in comparison to the core thickness, the error introduced is minimal. In this scenario, the size of the beam element is determined by the longitudinal size of the 2D plane stress solid elements for the core.

When the isotropic material is used for face-sheets, both beam elements and plane stress solid elements can be easily employed. However, for composite laminated face-sheets, the mechanical properties of the laminate should be provided or calculated before modeling it using beam elements.

2.2 Winkling of sandwich beam with isotropic face-sheets

Material and geometric properties of the sandwich beam are obtained from [6] and listed in Table 1. A 24ST clad aluminum alloy was used for the face-sheet and cork was used for the core. The sandwich specimens were constructed by bonding the facings to the core by means of primary and secondary glue [6].

Geometry or property	Value
Total thickness of sandwich beam, H (in)	1.0392
Width of sandwich beam, B (in)	2.0
Thickness of facing, t_f (in)	0.0196
Thickness of core, t_c (in)	1.0
Length of sandwich, L (in)	3.63
Elastic modulus of Aluminum facing, E_f (ksi)	9500
Poisson's ratio of Aluminum facing, v_f	0.25
Elastic modulus of Cork core, E_c (ksi)	1.18
<u>Poisson's ratio of Cork core, v_c</u>	0.136

Table 1. Geometric parameters and material properties [6]

FEA model convergence is checked first to determine the appropriate mesh and element size. Four-node bilinear plane stress elements CPS4 are used to model the core. While either quadratic beam elements B23 or bilinear plane stress elements CPS4 are used for the face-sheets. According to the convergence study 400 elements in the length direction and 80 elements in the thickness direction of the core are sufficient for accurate simulation. In order to capture the local effect at beam ends multi-point constraint (MPC) boundary conditions are created resulting for the end lines to behave as in contact with a rigid surface. The nodes at the end lines are allowed to move in the

thickness direction but are always constrained to remain in straight line connecting two extreme end nodes of the core. This straight line can stretch or shrink linearly and rotate around its central node. The translational freedoms of the node at the middle of the left end are constrained. The compressive load in the length direction is applied to the node at the middle of the left end with the transverse translational degree of freedom is constrained. The illustration of multi-point constraints and boundary conditions is shown in Figure 2. The critical load is then evaluated based on the smallest eigenvalue yielded by ABAQUS and the corresponding wrinkling mode can also be found.



Figure 2. Illustration of boundary conditions and load from testing

Initially beam elements B23 are used to model face-sheets then a study is carried to investigate the effect or using plane stress elements instead. When the MPC boundary conditions are applied at the ends the wrinkling mode of the sandwich specimen is shown Figure 3. In this scenario rotational degrees freedom (DOFs) of the nodes at both ends are not constrained. It can be found that the edge wrinkling occurs. FEA prediction is compared to that of testing in Table 2 indicating accurate prediction of wrinkling load. The actual wrinkling mode in the testing was edge wrinkling which can be observed by the deformation profiles of the specimen provided in [6].



Figure 3. Edge Wrinkling (MPC with enabled rotational DOFs of end nodes)

FEA (MPC with enabled rotational DOFs of end nodes)	Testing	Error (%)
373.89 lb/in	385.67 lb/in	-3.05

Table 2. Comparison between FEA edge wrinkling and test results from [6]

2.2.1 Effect of different boundary conditions

When the edge winkling mode is suppressed by releasing the MPC boundary conditions the wrinkling mode corresponding to lowest compressive load in the FEA model is shown in Figure 4. The compressive load in this situation is 498.84 *lb/in*. The wrinkling mode corresponds to span-wise wrinkling rather than edge wrinkling. If the rotational DOFs of end nodes are disabled the critical wrinkling mode is shown in Figure 5. The corresponding load when disabling rotational DOFs is 504.07 *lb/in*. The effect of the rotational DOFs is illustrated by comparison of both figures. In terms of critical load Table 3 provides a numerical comparison of FEA predictions in both cases. Corresponding winkling load when disabled rotational DOFs are higher. Meanwhile, the effect of rotational DOFs on span-wise wrinkling is insignificant.



Figure 4. Span-wise wrinkling mode with rotational DOFs enabled



Figure 5. Span-wise wrinkling mode with rotational DOFs disabled

Testing	Span-wise wrinkling	Span-wise wrinkling rotation disabled	
385.67 lb/in	498.84 (29.3 %)	504.07 (30.7)	-

Table 3. Comparison between FEA span-wise wrinkling and test results from [6]

The critical wrinkling load from testing in [6] is then compared with the analytical results in Table 4. It is clear that the FEA of edge wrinkling results are in better agreement with testing than that of the classical models (Winkler, Allen, Plantema, Hoff and Mautner, Niu and Talreja). It is also clear analytical predictions are closer to span-wise

wrinkling rather than edge wrinkling. Winkler model represents the closest solution to predict edge wrinkling among all classical solutions.

Model	Value (<i>lb/in</i>)	Error (%)
Winkler	474.43	23.0
Allen	553.27	43.5
Plantema	515.32	33.6
Hoff and Mautner	551.70	43.0
Niu and Talreja	566.13	46.8

Table 4. Comparison of classical analytical models with test results from [6]

2.2.2 Effect of different boundary conditions

This sub-section discusses the effect of alternate modeling of face-sheet where bilinear plane stress elements CPS4 are used to model the face-sheets instead of beam elements B23. The number of elements in the thickness direction of face-sheets is four with 400 elements along the length to match the core mesh. Similar to the case when using beam elements both edge wrinkling and span-wise wrinkling are investigated. The edge wrinkling mode associated to using MPC boundary condition in Figure 6. Span-wise wrinkling modes with rotational DOFs enabled or disabled are shown in Figure 7. Table provides the comparison between FEA predictions using bilinear plane stress elements CPS4 for the face-sheets to the test results [6]. Comparing percentage error from Tables 2, 3 and 5 indicates that bilinear elements slightly enhance edge wrinkling predictions. On the contrary, span-wise predictions are of less accuracy ~6% when compared to those obtained using beam elements.



Figure 6. Edge wrinkling using bilinear plane stress elements CPS4 for face-sheet



Figure 7. Span-wise wrinkling using bilinear plane stress elements (rotation enabled and disabled)

Testing	Edge wrinkling	Span-wise wrinkling	Span-wise wrinkling
295 67 lh/in	29/29(0.2.0)		
	384.38 (-0.3 %)	322.89 (33.0 %)	320.88 (30.0)

Table 5. Comparison between FEA wrinkling (face-sheets using CPS4 elements) and test results from [6]

2.3 Winkling of sandwich beam with laminated face-sheets

Sandwich panel with carbon fibre with vinylester face-sheets [19] are chosen for the comparison with FEA results. The face-sheet was laminated using four unidirectional layers in $[0/90]_s$ stacking sequence resulting in a total thickness of approximately 1 mm. In the test, carbon fibre laminated end tabs of 2 mm thickness and 25 mm width were bonded to the ends of the specimen. The specimens were placed between two plates one of which was fixed and the other was allowed to move to compress the sandwich panel. Divinycell H-grade foams were used for the core. Current study compares with specimens with H30 foam core. Specimen geometry and material properties obtained from [19] are shown Table 6.

Geometry or property	Value
Total thickness of sandwich beam, H (mm)	52
Width of sandwich beam, B (mm)	150
Thickness of facing, t_f (mm)	1
Thickness of core, t_c (mm)	50
Length of sandwich, L (mm)	200
Elastic modulus of laminated facing, E_1 (GPa)	107
Elastic modulus of laminated facing, E_2 (GPa)	15
Shear modulus of laminated facing, G_{12} (GPa)	4.3
Poisson's ratio of laminated facing, v_{12}	0.3
Elastic modulus of H30, E_c (MPa)	20

Table 6. Geometric parameters and material properties [19]

Bilinear plane stress elements CPS4 are used to model the laminated face-sheets. Four elements are used in the thickness direction of the face-sheet. Each element represents a lamina according to the laminate stacking sequence. The finite element model resembles the test setups with least simplification. The two tabs bonded in to the specimen in the experiment are modeled in the FEA model. The plates used to the compress the specimen in the test are modeled by two rigid surfaces. The predicted wrinkling mode from finite element simulation is shown in Figure 8. A mean critical load of 177.5 kN/m is reported from testing in [19]. Comparisons with FEA and classical analytical models are held and provided in Table 7. An error range over predicting the wrinkling load of 20% is identified in FEA, Plantema and Hoff and Mautner models. While Winkler under predicted the wrinkling load by 20%. The most accurate analytical solutions are provided by Allen and Niu and Talerja.



Figure 8. Wrinkling of sandwich panel with laminated face-sheet

Model	Value (kN/m)	Error (%)
FEA	220	23.9
Winkler	128	-27.9
Allen	183.15	3.2
Plantema	214.07	20.6
Hoff and Mautner	229.18	29.1
Niu and Talreja	190.18	7.1

Table 7. Comparison of classical analytical models with test results from [6]

3 DISCUSSION AND CONCLUSION

Analytical solution and finite element analysis are applied to investigate the wrinkling of sandwich structure. Two tests are obtained from literature, one with isotropic face-sheet and the second is with laminated face-sheets. In the finite element simulation the effect of boundary conditions and the element types of face-sheet are considered and compared. Also comparisons with classical analytical solutions are held to obtain better understanding of their predictions.

In FEA multi-point constraint (MPC) boundary conditions enabled accurate prediction of edge wrinkling. Also in the FEA the rotational DOFs of end nodes have slight effect on predicting span-wise wrinkling. Comparing to the sandwich with isotropic face-sheet concluded accurate prediction of edge wrinkling mode mainly using FEA. Meanwhile all classical analytical models predicted a higher wrinkling mode associated to span-wise wrinkling of the sandwich beam.

Comparing to the sandwich with isotropic face-sheet concluded large errors in predictions by FEA and most classical solutions. Despite accurate modeling used in the FEA of testing boundary conditions, end tabs and load application the wrinkling load is over predicted with 23.9% error. Some of the analytical solutions are in better agreement with test data than others. Meanwhile the error in analysis and simulation can be caused by imperfections in the test specimens. It is well known that imperfections have significant effect on stability characteristics and it is expected for a laminated face-sheet to possess larger imperfections than its isotropic counterpart.

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