

# FRACTIONAL VISCOELASTIC BEHAVIOR OF PREPREG WOVEN FABRICS

Sourki, R.<sup>1</sup>, Faal, R.<sup>1,2</sup>, Crawford, B.<sup>1</sup>, Vaziri, R.<sup>1\*</sup> and Milani, A.S.<sup>1\*</sup>

<sup>1</sup> Composites Research Network, The University of British Columbia, Canada

<sup>2</sup> Department of Mechanical Engineering, University of Zanjan, Iran

\* Corresponding co-authors ([abbas.milani@ubc.ca](mailto:abbas.milani@ubc.ca); [reza.vaziri@ubc.ca](mailto:reza.vaziri@ubc.ca))

**Keywords:** *Thermoplastic Woven Fabrics, Fractional Model, Viscoelastic Behavior*

## ABSTRACT

The energy dissipation during forming of composite materials may be rooted in various mechanisms. One of the main dominant factors is the partially viscous behavior of the resin, and even fibers, especially when they are intertwined. The viscous nature of these composite material components imply that their mechanical behavior should be modeled based on the rate of forming. In this article, the viscoelastic behavior of a typical thermoplastic woven fabric is obtained experimentally, both at room and high temperatures, and is subsequently modelled by a new method based on fractional calculus. The results clearly show that the viscoelastic moduli of the material, especially at lower temperature regimes, would be more accurately modeled through fractional time derivatives.

## 1 Introduction

To provide an accurate prediction of the forming processes of fabric composites at different forming conditions, comprehensive material models are required to capture the behavior of textile-reinforced fibers embedded in the surrounding matrix. Mostly in advanced forming numerical models, elastic material models, e.g. linear elastic, non-linear elastic models such as hyper-elastic and hypo-elastic [1-3] have been utilized for the reinforcing fabric. However, in practice, during forming processes a significant energy loss has been noticed which cannot be addressed with the above mentioned models. The energy dissipation can be due to several factors at the material level such as:

- Intra-yarn friction, crimping and undulation of fibers;
- chain disentanglement, rupture of bonds, network rearrangement, slippage and friction at micro-level between polymer chains of resin and fiber materials;
- friction between weft and warp yarns at meso-level;
- inter-ply friction, slippage and shear between different plies at macro-level;

Due to the above factors, hysteresis of composites can play a critical role during the pre-forming and forming stages, which is a key aspect of the analysis to capture the correct state of deformation and stresses at the end of the process. Bearing in mind that composite materials can experience a combination of tensile, shear and bending deformation modes during forming, it is essential to consider the dissipative effects during these modes. Also it should be mentioned that environmental effects such as processing

temperatures can affect the viscoelastic properties of composites [4]. Accordingly, the main motivation of the present study is to provide a new modelling framework, based on fractional calculus, to characterize the viscoelastic behavior of a textile reinforced composite at both room and high temperatures. Given the multi-scale nature of woven fabrics, the viscoelastic behavior both at meso-level (i.e. single yarns unraveled from the fabric roll) and at macro-level (i.e. the fabric ply) are studied herein. These results can be eventually used in the future process modelling framework to provide more accurate forming analyses.

## 2 Methodology

### 2.1 Experimental evidence for energy dissipation in a typical woven fabric

Figure 1 shows the tensile and bending behavior of a typical thermoplastic woven prepreg fabric used in this study under loading-unloading regimes to capture the hysteresis of the samples. The dominant energy loss in both in-plane and out-of-plane deformations is clearly evident. On the other hand, Figure 2 shows the material behavior as a function of time which is commonplace in forming processes, such as compression molding, stamp forming, RTM, VARTM, etc. [4,5]. It can be seen that the composite response varies notably at different time stations; 10-40% stress relaxation in less than 40 seconds at each step, depicting the dominant viscous behavior as the main source of the dissipative process.

### 2.2 Standard relaxation testing

Next, for the subsequent viscoelastic modeling purposes, standard stress relaxation tests were conducted where the sample was mounted on the gripper of an Instron tensile machine to apply tension up to 1.5% strain. The samples were then held undeformed for 10 hours at room temperature condition and then for 5 hours in a custom-designed thermal chamber at a prescribed temperature (95, 115, and 130 °C). The test setup is shown in Figure 3. In this process, temperature is controlled by a thermocouple with three wires. A cooling system as well as an extension bar was mounted onto the test-setup to prevent the temperature effect on the load-cell of the tensile machine.

### 2.3 Fractional viscoelastic modeling

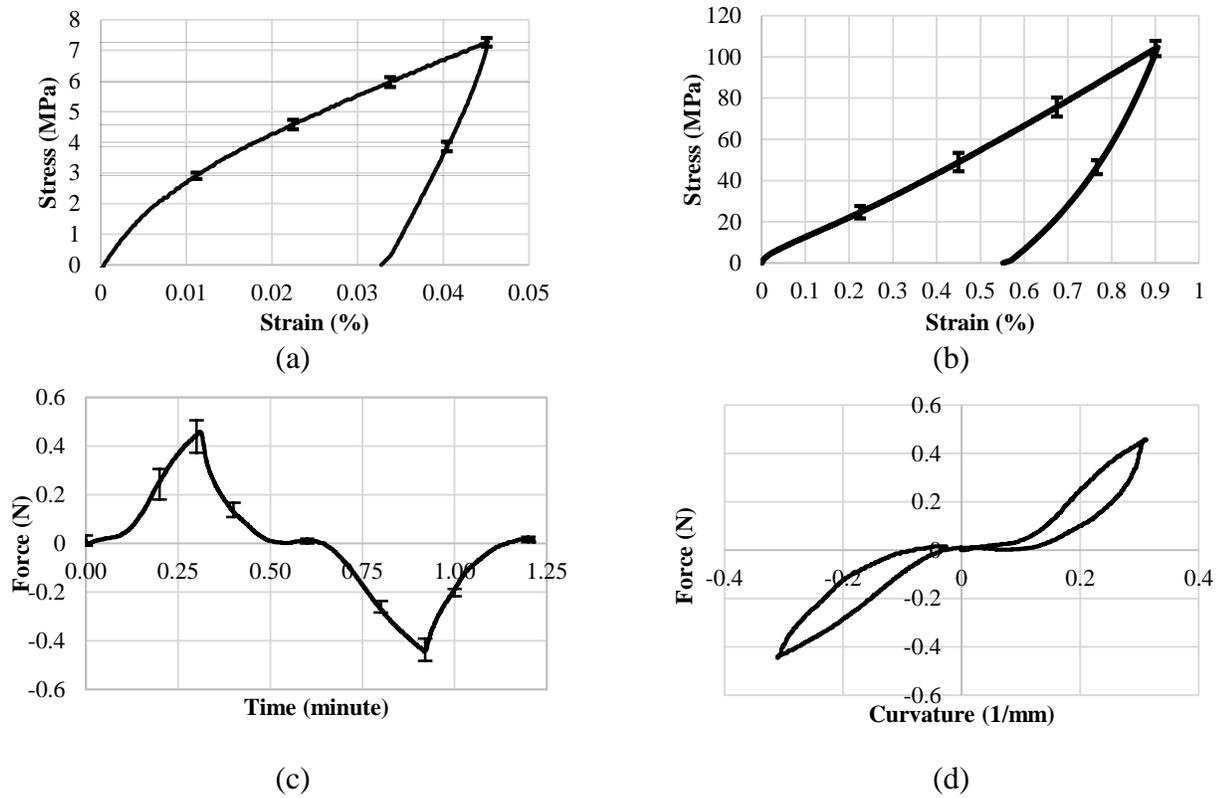
The relaxation modulus for the fractional derivative model used in this study can be written as follows [6]:

$$G(t) = \frac{\sigma(t)}{\varepsilon_0} = E_0 \left\{ 1 + \left[ \left( \frac{\tau_\varepsilon}{\tau_\sigma} \right)^\alpha - 1 \right] E_\alpha \left( - \left( \frac{t}{\tau_\sigma} \right)^\alpha \right) \right\} \quad (1)$$

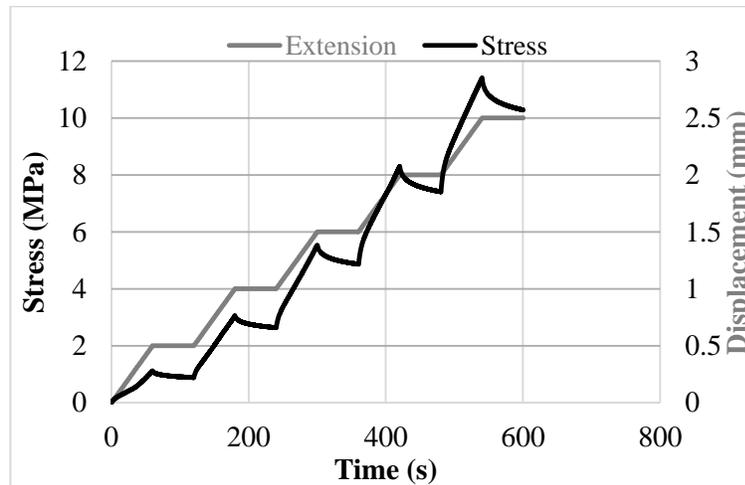
in which  $E_\alpha(\cdot)$  is the Mittag-Leffler function of order  $\alpha$  given by

$$E_\alpha(x) = \sum_{n=0}^{\infty} \frac{x^n}{\Gamma(1+n\alpha)} \quad (2)$$

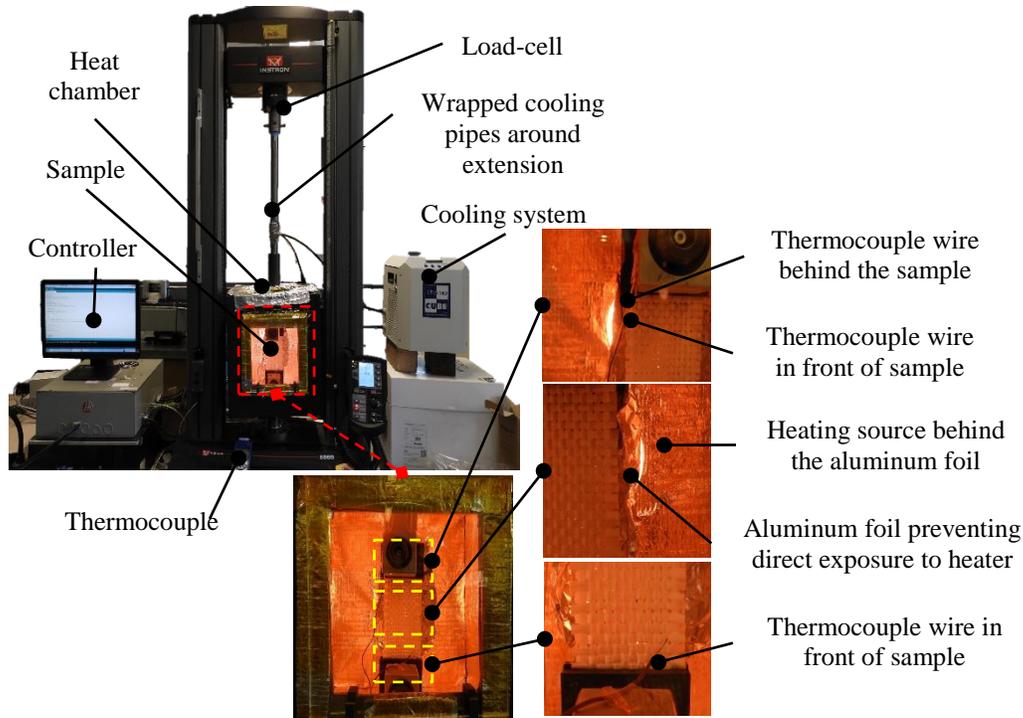
where  $\Gamma(\cdot)$  is the Gamma function. In this model, there are four parameters  $\alpha$ ,  $E_0$ ,  $\tau_\sigma$  and  $\tau_\varepsilon$  which can be identified for the fabric from the standard relation experiments. It is worth mentioning that the case  $\alpha = 1$  represents the model at the integer order, resulting in  $E_\alpha(x) = e^x$  and hence the stress response in the form of  $\exp(-t/\tau_\sigma)$  for relaxation.



**Figure 1:** Conducted tests under loading-unloading regimes of an E-glass fiber/ polypropylene pre-consolidated plain weave (under commercial name Twintex), depicting a large dissipative energy at both deformation modes: (a) low strain response for tensile test, (b) high strain response under tensile test, (c) reaction force vs. time for bending experiments and (d) the force-curvature curve during cyclic bending experiments. Each test was repeated three times.



**Figure 2.** Response of the sample under a step-wise forming regime.



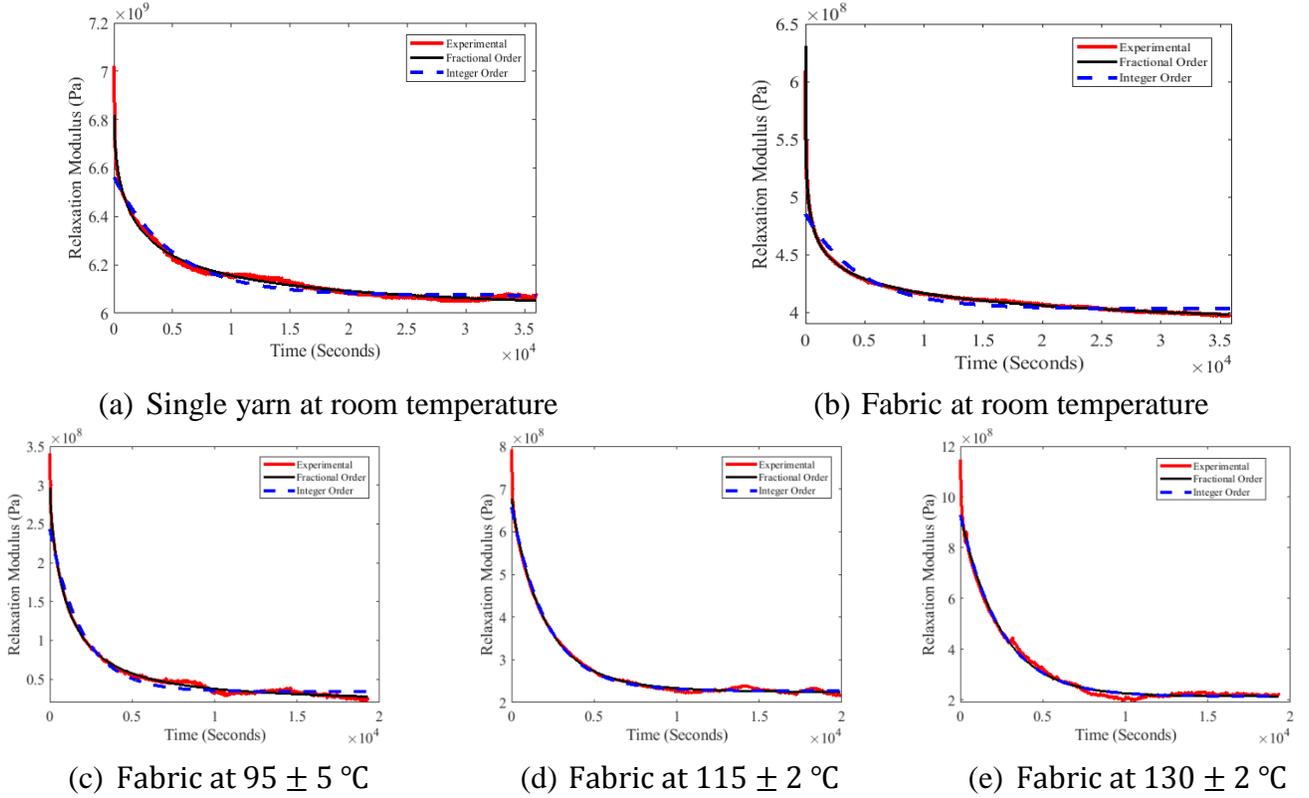
**Figure 3.** Experimental setup employed in the relaxation tests.

### 3 Results and Discussion

In this section, viscous parameters of a single yarn at room temperature, fabric at room temperature as well as 95, 115, and 130 °C are presented. Two approaches, fractional order (i.e. the new model) and the standard integer order (i.e., standard linear solid model also known as Zener model [6]) are examined to compare and analyze the results. The calibrations of the models are carried out in MATLAB by minimizing the error between the experimental data and the predictive model. As can be seen from Figure 4, the fractional derivative model shows a more accurate calibration compared to the integer model. The most striking observation from this data comparison was that at higher temperatures, just before the melting temperature range (110~175 °C), the predictions of the fractional model and the integer model are similar. At this stage, the polymer matrix enters the melt state, and starts to flow, a behavior that was also observable with naked eye. In other words, the effect of the polymer (mainly responsible for the rubbery-like behavior of the fabric) reduces at high temperatures and the composite fabric enters a glassy regime ( $\alpha \rightarrow 1$ ) dominated by the fiberglass fabric reinforcement and only then, the integer modeling (as commonly employed in the literature) becomes sufficient to represent the fabric composite material's behavior. The identified model parameters are summarized in Table 1.

**Remark:** Recently the authors have also compared the performance of the fractional model with the frequently used Prony series for the composite yarn (see [7] for details). As expected, it was found that the Prony series can show far better results than the integer models. However, by comparing 2-term and 3-term Prony series to the fractional model, it was found that the prediction error evaluations are much favored toward the fractional model. It is worth highlighting the fact that the fractional model has only used 4

terms in comparison to the 2-term and 3-term Prony series, which have a total of 4 and 6 parameters, respectively.



**Figure 4:** (a) Relaxation of a single yarn and the fabric at different forming temperature conditions: predictive models vs. the experimental data from relaxation tests.

**Table 1:** Parameters of the two models for the single yarn and the fabric samples tested

Temperature	$\alpha$	$\tau_\sigma$	$\tau_\epsilon$	$E_0$ (GPa)
<b>Single yarn</b>				
Fractional model				
Room temperature	0.571	2884.5	3669.3	5.616
Integer model				
Room temperature	1	49655.8	5364.7	5.735
<b>Fabric</b>				
Fractional model				
Room temperature	0.3474	490.20	$2.4909 \times 10^3$	0.35860
95 ± 5 °C	0.7530	1419.9	$7.3727 \times 10^4$	0.01515
115 ± 2 °C	0.9418	2037.3	$6.7269 \times 10^3$	0.21985
130 ± 2 °C	1.0024	2445.9	$1.0547 \times 10^4$	0.21463
Integer Model				
Room temperature	1	4557.3	5483.3	0.40344
95 ± 5 °C	1	1968.4	$1.4054 \times 10^4$	0.03407
115 ± 2 °C	1	2156.0	$6.2298 \times 10^3$	0.22749
130 ± 2 °C	1	2441.1	$1.0606 \times 10^4$	0.21406

## 4 Conclusion

During composite forming processes, the prepreg fabrics undergo a mixture of combined loading and displacement boundary conditions, e.g. in-plane and out-of-plane deformation, multi-step deformation, along with severe thermal and pressure effects, loading-unloading, etc. During these processes, irreversible behavior of the fabric, even at room temperature, is observed which affects and alters properties of the final components. Hence, the viscoelastic behavior of a select woven fabric, as the main source of dissipative process during forming, was studied and analyzed by means of an improved modeling framework based on fractional derivatives. The model consisted of four unknown parameters that were determined by fitting the models to the experimental data from relevant tests and compared to the classical integer model. The results indicated that the fractional model has far more accuracy in terms of characterizing the viscous behavior of the material, especially at the early stages of relaxation. The most interesting finding was that at higher temperatures (where the polymer matrix became highly compliant), the fractional model showed a more reinforcement (fiberglass)-dominant behavior ( $\alpha \rightarrow 1$ ) for the fabric composite viscoelasticity and the prediction became comparable to the integer model.

## 5 Acknowledgment

The authors wish to acknowledge the financial support from the Natural Sciences and Engineering Research Council (NSERC) of Canada. The support from colleagues at the Composites Research Network is greatly acknowledged.

## References:

1. Milani AS, Nemes JA. An intelligent inverse method for characterization of textile reinforced thermoplastic composites using a hyperelastic constitutive model. *Composites Science and Technology* 2004;64(10):1565-1576.
2. Komeili M, Milani AS. Finite Element Modeling of Woven Fabric Composites at Meso-Level under Combined Loading Modes. In: *Advances in Modern Woven Fabrics Technology*. Rijeka: IntechOpen, 2011. Ch. 4.
3. Charmetant A, Orliac JG, Vidal-Sallé E, Boisse P. Hyperelastic model for large deformation analyses of 3D interlock composite preforms. *Composites Science and Technology* 2012;72(12):1352-1360.
4. Campbell FC. *Manufacturing Processes for Advanced Composites*. Elsevier Science & Technology, 12.
5. Strong AB. *Fundamentals of Composites Manufacturing: Materials, Methods and Applications*. Society of Manufacturing Engineers, 2008.
6. Caputo M, Mainardi F. A new dissipation model based on memory mechanism. *Pure Appl Geophys* 1971;91(1):134-147.
7. Teymori Faal R, Sourki R, Crawford B, Vaziri R, Milani AS. Using fractional derivatives for improved viscoelastic modeling of textile composites. Part I: fabric yarns (under review).