

# BISTABLE COMPOSITES: STABILITY CHARACTERISTICS AND ACTUATION REQUIREMENTS

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## ABSTRACT

Thin unsymmetric composite laminate when cured flat in an autoclave is found to possess two equilibrium shape, hence called bistable panel. Under the effect of external disturbance, a bistable panel undergoes a sudden jump from one equilibrium shape to another non-adjacent equilibrium shape. This snapthrough behavior is concluded to be advantageous for the design many applications requiring shape morphing capabilities in which snapthrough is triggered by using actuators of different types, e.g., piezo actuators. In these applications, accurate assessment of actuation requirements is essential for guaranteeing optimal performance and preserving bistable characteristics of the panel. This work investigates the effect of geometry migration on strain energy requirements to trigger snap-through behavior. For this purpose, the effects of shape scaling of square panels and aspect ratio of rectangular ones were studied. Strain energy density (SED) was employed to assess relative equilibrium of bistable shape and actuation requirements. For a square panel SED associated to non-adjacent equilibrium shapes is almost equal. On the other hand, in the case of a rectangular panel, there exists one global minimum of the SED which is associated to the primary equilibrium shape, while the secondary equilibrium shape exists at a local minimum of the SED.

Based on analyzing SED in terms of equilibrium position and actuation requirements bistable panel geometry, i.e. aspect ratio, was identified. Moreover, bonding method of piezoelectric actuators to panels' surfaces was assessed. Conclusions were drawn regarding optimizing the shape of actuated panels and actuators bonding technique.

## 1 INTRODUCTION

Initially Hyer [1] documented his observations on cured shapes of thin unsymmetric laminates when cured flat in an autoclave. Ergo, a thin unsymmetric cross-ply laminate when cured possesses two stable equilibrium cylindrical shapes with equal, opposite and orthogonal curvatures as shown in Figure 1. Under external effects, e.g. force, this laminate can be triggered to snapthrough from one equilibrium shape to another. Meanwhile, the Classical Lamination Theory (CLT) could not be used to predict room-temperature shapes of unsymmetric laminates. Thus, Hyer's extended the CLT [2, 3] based on the Principle of Minimum Total Potential Energy (PMTPE) and accounting for geometric nonlinearities to successfully predicted room-temperature shapes of thin unsymmetric laminates. For the following two decades Hyer and his colleagues continued to lead the research community by providing valuable in-depth expositions on unsymmetric laminates and their bistable behavior. For example, Akira and Hyer [4] and Ochinerio [5] provided evidence on the existence of geometric imperfections investigating their effect on cured shapes of unsymmetric laminates. Furthermore, Hyer and his colleagues proposed using bistable panels in morphing applications due to their favorable energy saving characteristics. For this purpose they investigated various methods

of actuation, prominently the work by Dano [6] adopted Shape Memory Alloys (SMA) as means for triggering snapthrough behavior. Correspondingly, Schultz and Hyer [7] successfully used piezoelectric actuators to trigger snapthrough behavior.

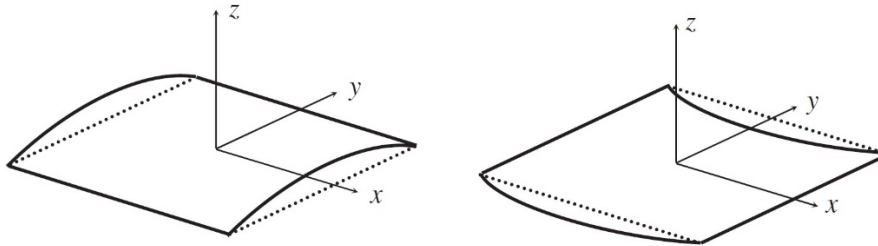


Figure 1. Stable shapes of thin unsymmetric cross-ply laminate

Therefore, bistable laminates found their way to wide range of applications in a variety of fields among which are bionics and energy harvesting [8]. A majority of applications were developed in Aerospace engineering [9-12]. A central aspect in all these applications was actuation or driving technique responsible for triggering shape change. A variety of techniques were investigated and proved successful, namely, SMA, flexible piezoelectric, thermal, and magnetic actuators [13-19]. Without a doubt, these methods were proven effectively however there were comparisons held among these actuation techniques. In other words, actuators were proven to function meanwhile were not proven to be optimal.

Therefore, this work proposes utilizing the Strain Energy Density (SED) to assess optimal performance of actuators. For this purpose, a number of bistable laminates with different geometries were examined. Effects related to change of geometric parameters were studied by examining the total SED of the panels. This consistent study delineated two major aspects with the potential to be employed to identify optimal actuation technique and specific actuator's functionality. To conclude, a successful case of an actuated bistable panel was examined.

## 2 METHODS

As indicated earlier geometric imperfections were observed in an unsymmetric panel and they were dominantly responsible for the bistable behavior [4,5]. Tawfik et al. [21] developed a Finite Element Analysis (FEA) methodology that accounts for imperfection. This methodology was employed [22-24] to predict room-temperature shapes as well as actuation requirements of bistable laminates. Nakhla and Elruby [25] further developed this methodology elucidating its correlation with Koiter's asymptotic postbuckling theory [26]. The general procedure provided in [25] was used to study the design space of Uninhabited Aerial Vehicle (UAV) morphing wing concluding valuable insights on the design parameters of a Hyper-Elliptic Cambered Span (HECS) wing. This general procedure is adopted in current work to allow detailed study of SED with the purpose of pursuing optimal actuation.

The general procedure [25] consists of two modules, namely, curing simulation and snapthrough analysis. In the curing analysis, geometric imperfections are obtained and utilized to perform geometrically nonlinear analysis to predict room-temperature shapes of a bistable laminate. While in the snapthrough analysis nonlinear analysis based on Riks theory [27] is utilized to predict actuation requirements. In [25] a simple force normal to the laminate was used to trigger snapthrough behavior. Meanwhile, the methodology [18] and general procedure [25] can be adapted to account for different actuation techniques. For further details, the reader is encouraged to consult with [23,25].

For the purpose of calculating SED it is sufficient to employ a force to trigger snapthrough behavior. Figure 2 provides a schematic to illustrate the general procedure together with the SED concept. Starting from the equilibrium point shown to the left on the deflection axis, a positive concentrated load  $P$  is applied at the center of the panel in nonlinear analysis step. The applied load is incremented until  $P \geq P_{cr}$ , or until the laminate is deformed beyond its second equilibrium shape. The slope of load deflection ( $P$ - $\delta$ ) curve is monitored as load increases changing from positive slope to a constant plateau. As the load is further increased the  $P$ - $\delta$  curve abruptly gains positive slope and the laminate is deformed beyond its second equilibrium configuration. At that point a no nonlinear unloading step is conducted to allow the laminate to reach its second equilibrium configuration. Once the second equilibrium configuration is reached, the loading/unloading nonlinear analysis steps are repeated using a concentrated load in the opposite direction. Throughout this manuscript the term ‘snap-through’ will be used in reference to laminate shape change from its first to second equilibrium position. Meanwhile, the term ‘snap-back’ is used in reference to the opposite process; i.e. from second to first equilibrium configuration.

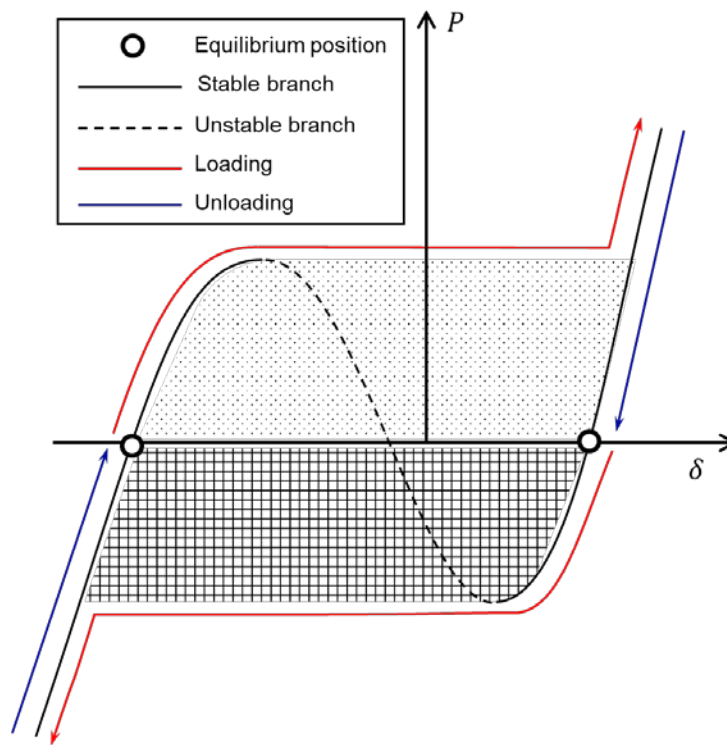


Figure 2. Schematic of the general procedure and strain energies for snapthrough analysis

The dotted area under the  $P$ - $\delta$  curve represents the Strain Energy required to trigger snap-through. While the cross-hatched area under the  $P$ - $\delta$  curve represents the Strain Energy required to trigger snap-back. Calculating these areas would provide knowledge on actuation requirements in terms of required work output. Therefore the following section is dedicated to discuss relative stability of equilibrium positions and the required strain energies (or work done) to trigger snapthrough behavior. In brief, the challenging task of any designer is utilize an efficient actuation technique while maintaining the laminate’s bistable configurations. Particularly, flexible piezoelectric actuators are appealing meanwhile their stiffness is yet relatively high. Therefore, bonding these actuators to a bistable laminate would potentially change its bistable characteristics.

### 3 RESULTS AND COMPARISONS

A number of cross-ply laminates with various geometry are chosen to study the effect of varying their geometry on their stable configurations and their actuation requirements. Graphite-epoxy IM7/8551-7 materials properties were adopted throughout this section and provided in Table 1. All laminates possess  $[0_2/90_2]$  stacking sequence unless mentioned otherwise.

Table 1. Material properties of IM7/8551-7 graphite/epoxy prepreg

$E_{11}$ (GPa)	$E_{22}$ (GPa)	$G_{12}$ (GPa)	$\nu_{12}$	$\alpha_1$ ( $10^{-6}/^\circ\text{C}$ )	$\alpha_2$ ( $10^{-6}/^\circ\text{C}$ )	$t$ ( $\mu\text{m}$ )
146.14	8.472	3.879	0.341	0.14	30.98	138.75

Initially, the effect of varying the side length on stability characteristics of square laminates is studied, i.e. geometry scaling. The side length is changed in increments of 25 mm from 75 mm to 150 mm.

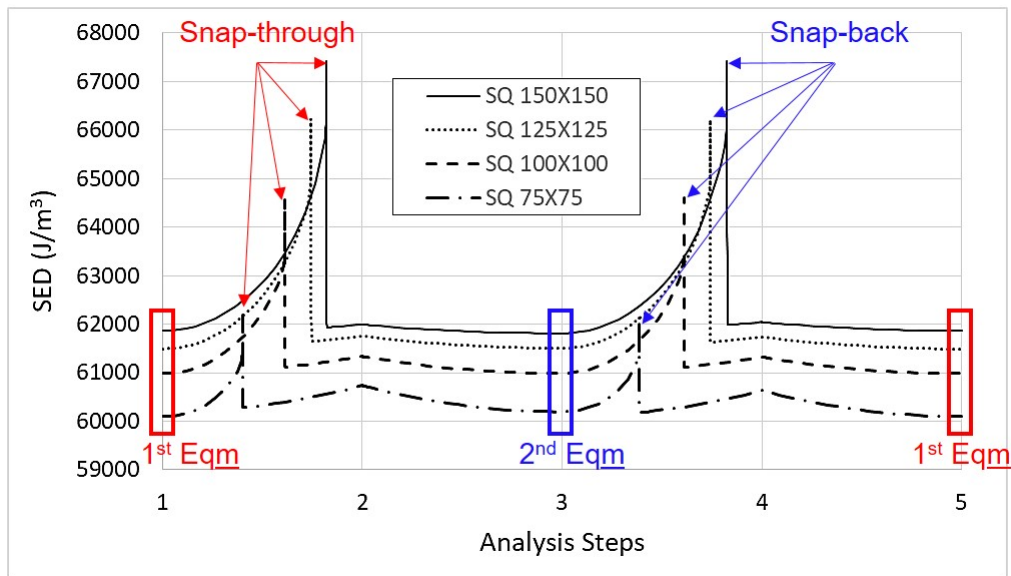


Figure 3. SED of square laminates

In Figure 3, the loading steps are 1 → 2 and 3 → 4 while 2 → 3 and 4 → 5 are the unloading steps. A concentrated load of magnitude 10 N was used to trigger snap-through and snap-back. Therefore it can be seen that reducing the side length results in reducing force requirements on snap-through/back loads. It can be easily observed that due to the equal sides of a square the magnitude of required force to trigger snapthrough behavior is almost identical. It can also be observed the positions of equilibrium, first and second, occur at almost equal absolute SED values. Reducing the side length results in consistent reduction in SED levels of both equilibrium shapes. Thus in a square panel, the side length is in direct correspondence with the SED value of the equilibrium position as well as the actuation requirements.

To further investigate the effect of geometry migration, the stability requirements of rectangular laminates were studied. One side is maintained at 150 mm while the other side was reduced to 50 mm in increments of 25 mm. Figure 4 is constructed with similar notations to Figure 3; i.e. equilibrium shapes are at positions 1, 3 and 5 of the horizontal axis.

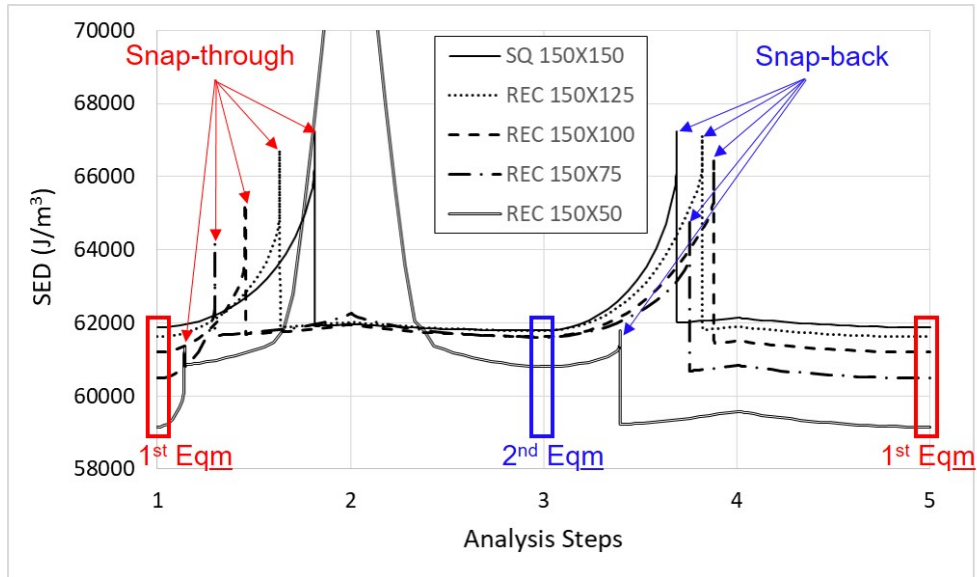


Figure 4. SED of rectangular laminates

It is clear that reducing length of one side has similar effect on the first equilibrium configuration and snap-through load as in the square laminates. Meanwhile, as one side length is reduced (aspect ratio is increased) the snap-back load is initially increased (in laminates  $150 \times 125 \text{ mm}^2$  and  $150 \times 100 \text{ mm}^2$ ) then decreased. This stiffening effect was originally documented in [23] and is related to shallowness of the panel along the shorter side of the rectangle. Meanwhile, the total SED of the second equilibrium shape is barely affected till the side length is reduced below 75 mm. In the meantime, the first equilibrium shape continued to possess absolute minimum value of SED hence it can be referred to as the primary shape. The case of  $150 \times 50 \text{ mm}^2$  demonstrated consistent effect on the primary equilibrium shape and the snap-through force. Also, this aspect ratio resulted in significant reduction in bending stiffness, i.e. the overshoot of SED at position 2 on the horizontal axis. In reality the rectangular laminate in this case exhibit the behavior of same sense bending of tape springs [28]. In case of further reducing the side length the laminate would lose its bistable configuration, i.e. maintain only its primary equilibrium shape being curved along the longer side, this observation was also originally documented in [23]. It is clear that observations obtained by assessing the SED are consistent with all concepts documented earlier in literature. Thus, the next step is to demonstrate utilizing SED to assess actuator performance and its influence on bistable characteristics.

In [18,23] they successfully utilized Macro-Fiber Composite (MFC) actuators. Given that they were first to identify the stiffening effect in rectangular panels, they proposed attaching actuators to the top and bottom of the panel with them being both along the shorter side of the panel. This suggestion was to fully utilize the stiffening effect as well as preserve bistable characteristics of the panel. To preserve bistable characteristics, they proposed bending the actuators to conform to panel's curvature prior to bonding into one assembly. In doing so they successfully used significantly smaller MFC actuators. Figure 5 provides equilibrium shapes of their bonded actuators/panel assembly.

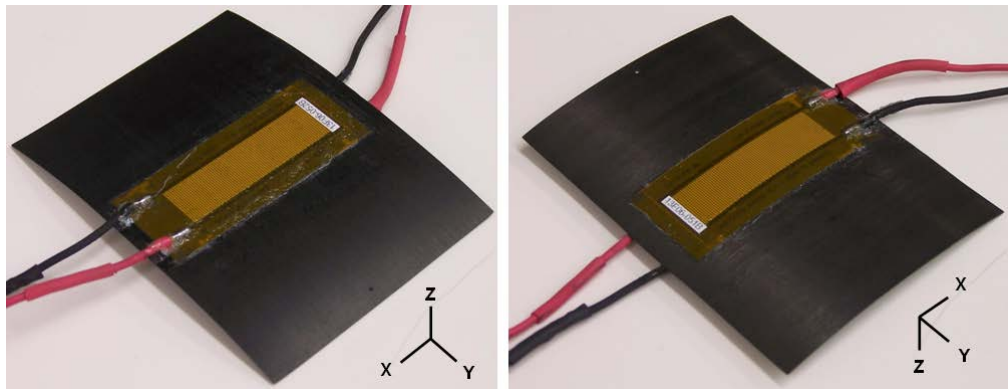


Figure 5. Bonded actuators/panel assembly [18,23]

The general procedure [25] is used to replicate the FEA model developed in [18,23]. In order to ensure accurate FEA model development the voltage requirements for snapthrough behavior was verified against experimental results [18,23]. Meanwhile, actuator functionality is not the main focus of current work. Instead, actuation assessment is sought to provide insight on the utilization of any actuators and associated effects on bistable characteristics. Therefore, upon verifying FEA model accuracy the SED was obtained for the bistable panel before and after bonding the actuators. Figure 6 compares SED of panel in both cases utilizing a concentrated load to trigger snapthrough behavior.

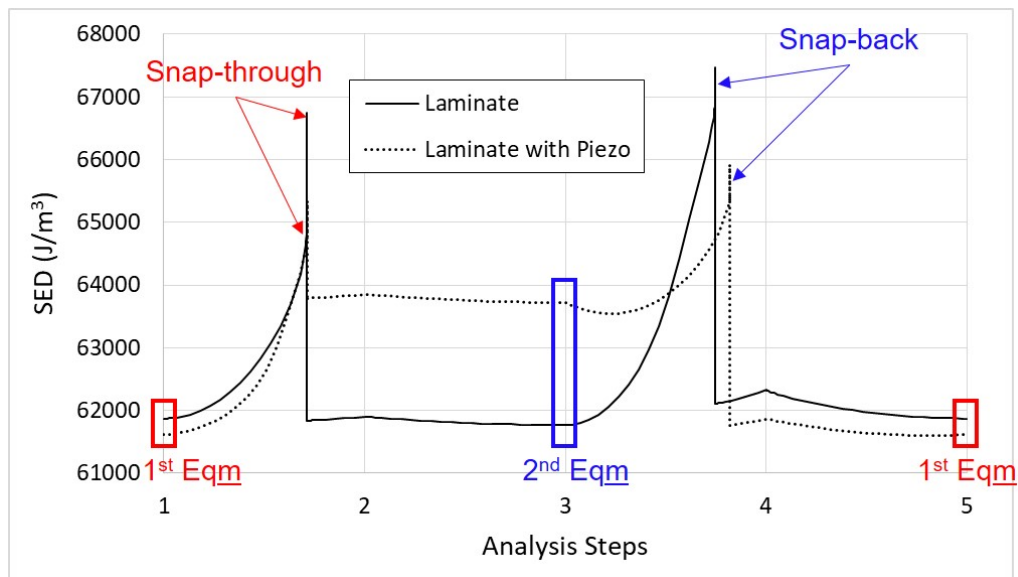


Figure 6. SED of rectangular laminate before and after actuator bonding

It can be observed in Figure 6 the first equilibrium shape was slightly affected by bonding of actuators. Meanwhile, the second equilibrium shape is significantly affected displayed large increase in its SED value due to actuators bonding. Therefore, the first equilibrium shape possessed the absolute minimum SED position of the panel hence becoming its primary equilibrium shape. This also implies that the second equilibrium shape is indeed in jeopardy of being eliminated and justifies proposing actuators bonding to the panel in its curved configuration. In reality,

actuators bonding possess similar effect on snapthrough characteristics to reducing side length of a rectangular laminate. This is supported by noticing that bonding the actuators resulted in increasing the load required to trigger snap-back. Finally, while MFC actuators were successfully utilized to trigger snapthrough behavior [18,23] they may have not been optimally employed as they adversely affected the stability of second equilibrium shape. Alternative scenarios of actuators bonding could have been assessed, e.g., both actuator aligned with longer side or orthogonal actuators, one parallel to each side of the panel.

## 4 CONCLUSIONS

Initially consistent study of shape scaling and shape migration of bistable laminates was conducted. Throughout the study, SED was used as quantitative tool for both snapthrough requirements (critical loads) and relative stability of cured configurations or shapes. Later, SED was used to assess usage of MFC actuators to trigger shape change of rectangular bistable laminate. Interrogating SED results confirmed success of the originally proposed actuation bonding technique, i.e. bonding to curved configuration. Furthermore, SED results documented the adverse effect on one of the stable shapes of the panel that were mainly caused by actuator bonding. In conclusion, SED proven to be an efficient quantitative analysis tool for the assessment of actuation method and optimal actuator utilization.

Further results will be available in CANCOM 2022 presentation.

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