

Vacuum-Infused PMMA-Cotton Composite Textile with Tunable Mechanical Properties: A Multivariate Analysis

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1. Abstract

Natural-fiber textiles continue to attract the apparel industry for their lightweight characteristics, comfort, and hand-feel; yet their hydrophilicity, porosity, and weak fiber-matrix interfaces limit their structural performance. In this work, to evaluate whether dilute impregnation can densify and strengthen the fiber architecture without sacrificing formability, a typical plain-woven cotton fabric was vacuum-infused with acetone-borne poly(methyl methacrylate) (PMMA) at 0, 0.1, 0.5, 1.0 and 5.0 wt.% and dried to form PMMA–cotton composites. Uniaxial tensile testing was performed on all material formulations, and the property response was found to vary non-monotonically with PMMA loading: stiffness and ultimate strength first decreased at 0.1 wt.%, recovered between 0.5 and 1.0 wt.%, and then deteriorated again at 5.0 wt.%, ruling out a simple loading–property scaling. Differential scanning calorimetry confirmed amorphous PMMA incorporation with glass-transition midpoints at 100–108 °C and preservation of the cellulose endotherm near 142 °C. For modeling, a ten-descriptor material property envelope (E , σ_u , ϵ_u , ϵ_b , U , V_f , ϕ , θ , m , R_w ; see the nomenclature) was compressed by Principal Component Analysis (PCA) onto two orthogonal axes capturing 67.7% of the variance; surpassingly, the first principal component (PC1) clearly emerged as a strength–toughness–reliability axis and the second principal component (PC2) as a stiffness–wettability axis. Ward-linkage hierarchical clustering and k-means partitioning were employed, and they converged on three material response regimes: under-bonded (between 0-0.5 wt.%), engineering (functional) window (between 0.5-1.0 wt.%), and saturated (>1.0 wt%). A small-sample two-parameter Weibull screening (bootstrap 95% CIs) complemented a porosity-penalized rule of mixtures and a Kelly–Tyson bridge model that reproduced the design envelope. Overall, results identified that 1.0 wt.% PMMA loading as an engineering optimum for the new material system, providing the highest ultimate strength ($\sigma_n = 67.4$ MPa, +3% over the untreated control), the highest toughness ($U = 8.9$ MJ m⁻³, $\sim 4\times$ over the control), the largest rupture strain ($\epsilon_m = 23\%$), the tightest within-set strength scatter ($CoV \approx 0.5\%$), 89% post-laundry strength retention ($R_v = 89\%$) and a contact angle of $\theta \approx 76^\circ$ consistent with preserved wettability.

2. Introduction

Cotton is among the largest natural fibers used in apparel and soft-goods manufacturing, prized for its comfort, breathability, hand-feel, and low embodied-energy profile [1], [2]. According to the Textile Exchange Materials Market Report 2025, cotton accounted for ~19% of the 132 Mt global fibre output in 2024 (~24.5 Mt), making it the second-most-produced fibre after polyester (~59% share, of which ~88% is fossil-based) [3]. Cotton's share has been eroding year-over-year as polyester volumes continue to climb [3]. Retaining a natural cotton main component rather than substituting it outright with nylon or polyester is therefore desirable for sustainability, biodegradability, and consumer-perceived comfort, motivating the present strategy of strengthening cotton in place rather than replacing it with a synthetic backbone.

Transitioning cotton fabrics to structural or semi-structural elements, however, exposes three longstanding limitations: hydrophilicity that promotes moisture uptake and dimensional instability [4]; high intrinsic porosity that reduces the load-bearing capacity [1]; and poor fiber–fiber cohesion within the weave, leading to progressive pull-out and premature failure [5]. Upgrading the material to such elements requires a design strategy that reinforces micro-scale features without sacrificing the fabric's flexible architecture/drape. Two broad routes dominate the current literature in this regard: neat resin impregnation, which converts the base natural fiber fabric into a rigid ply but largely eliminates hand-feel and drape [4]; and ultra-dilute surface treatments, which preserve drape but deliver only marginal reinforcement [5]. The intermediate regime—using deliberately low yet structurally meaningful polymer content -is hypothesized to produce non-monotonic property responses with the impregnator loading (e.g., stiffness and ultimate strength can drop at sub-critical loadings, recover at intermediate loadings, and decay again under saturation), such that the optimum design cannot be identified from any single scalar property and instead requires the joint, multivariate consideration of several mechanical, wettability and reliability descriptors. Combined with classical composite mechanics [6], [7], the present work aims to employ select machine learning techniques [8]–[10] towards such optimum design of dilute-impregnated PMMA–cotton composite, as a case study.

PMMA was selected as the impregnator on the basis of four converging arguments, the first of which is a sustainability/circular-materials argument anchored to current production-and-waste statistics. Globally, PMMA is produced at ~3.9–4.0 Mt yr⁻¹ (USD ~5 billion market) [11], yet only ~10% of post-consumer PMMA is recycled, with the remainder landfilled or incinerated [11], [12]; in Europe alone the polymer generates ~30 kt of waste per year [11]. In Canada, the textile and apparel sector itself is a major plastic-waste contributor: ~290 kt of synthetic textile products (polyester, nylon, PVC and acrylics the family that includes PMMA) entered the Canadian market in 2020 and ~280 kt were disposed of the same year, of which ~98% ended up in landfill, ranking textiles the fifth-largest plastic-waste category in the country [13], [14]. Diverting even a small fraction of this thermoplastic stream into a non-permanent reinforcement of natural-fiber fabrics — rather than the conventional thermoset-based composite route can offer a measurable circular-materials benefit (recovered PMMA can in principle be re-dissolved, re-precipitated or thermally depolymerized to monomer) and motivates the present route.

Beyond this circular-materials argument, PMMA is also technically matched to cotton: (i) it is an amorphous, optically transparent thermoplastic with a glass-transition temperature ($T_g \approx 105^\circ\text{C}$) above typical apparel service temperatures yet well below cellulose decomposition, allowing inter-fiber bridges to be formed without thermally damaging the cotton [15]; (ii) it is fully soluble in acetone, enabling low-viscosity vacuum impregnations and clean solvent removal; (iii) it has been used previously to reinforce bacterial- and nano-cellulose, methylcellulose, and natural-fiber substrates with reported tensile-modulus and tensile-strength gains of 1.6–1.8 $\times$ at sub-1 wt.% loadings [16]–[18], providing direct precedent for the dilute-loading window proposed here; and (iv) it is recyclable and chemically compatible with the dominant downstream finishing chemistries used in apparel manufacturing.

For machine learning-based material design, a ten-descriptor envelope of the PMMA-cotton composite properties is projected onto a latent space, clustered without supervision, and reconciled with a porosity-penalized rule of mixtures [7] and a Kelly–Tyson bridge model [6]. Reliability of the material performance is quantified by a two-parameter Weibull distribution [19], wettability variation via the Cassie–Baxter framework [20], and thermal response against the Gaur–Wunderlich PMMA baseline [15]. Mechanical and wash fastness tests employed include uniaxial tension (ASTM D5034-21) [22] and laundering protocol AATCC 61-2A [23], as described next.

3. Materials and methods

3.1. Sample preparation

Plain-woven, commercially-available 100% cotton fabric (areal density $145 \pm 3 \text{ g m}^{-2}$, thickness $0.62 \pm 0.04 \text{ mm}$) was selected to represent a broadly representative apparel textile and, after conditioning at $23^\circ\text{C} / 50\% \text{ RH}$ for 24 h, was uniformly treated with a low-viscosity, acetone-borne PMMA solution (PMMA $M_w \approx 120 \text{ kg mol}^{-1}$, dispersity $\bar{D} \approx 2.0$).

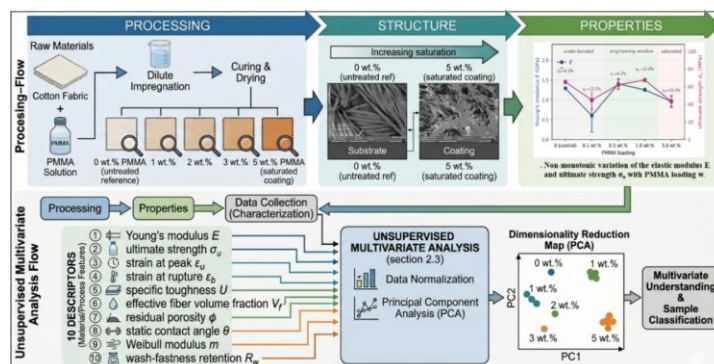


Figure 1. Processing–structure–property flow for dilute-impregnated cotton–PMMA hybrid textile samples prepared. Five PMMA loadings (0–5 wt.%) sweep the substrate from an untreated reference to a saturated coating; ten descriptors (material/process features) feed an unsupervised multivariate analysis (section 2.3).

For impregnation, five PMMA weight fractions were considered $w = 0, 0.1, 0.5, 1.0$ and $5.0 \text{ wt.}\%$ relative to the dry mass of the fabric, to sweep from an untreated reference to a saturated coating. The impregnation was performed in a sealed vessel with a reduced-pressure stage at 10 kPa absolute; solvent was then removed by a 12 h drying step at room temperature, and specimens were conditioned at $23^\circ\text{C} / 50\% \text{ RH}$ for a further 48 h before any further characterization. Figure 1 summarizes the employed processing–property flow from substrate selection through

multivariate analysis. The procedure was deliberately designed to use a low-viscosity (acetone-borne) solution and a vacuum-assisted rather than mechanically forced — infusion stage, so that capillary action drove the PMMA preferentially towards inter-fiber contact points without redistributing the warp/weft yarns or distorting the plain-weave geometry; alternative routes such as hot-pressing, padding-rolling, or high-viscosity neat-resin impregnation were avoided because they would have flattened the yarn cross-sections and partially closed the inter-yarn channels, masking the dilute-loading effect under investigation [5].

3.2. Characterization

Areal density, thickness, and fiber volume fraction values were characterized gravimetrically and geometrically. Specimen morphology was investigated at multiple magnifications where both yarn-scale coverage and fiber-scale bridges could be observed using a scanning electron microscope/SEM. Uniaxial tensile experiments followed ASTM D5034-21 [22], where fabric coupons (rectangular fabric strips per the ASTM D5034 standards, gauge 100×25 mm) were tested (using an Instron universal tensile machine) under displacement rate of 50 mm min^{-1} , with engineering stress obtained from the as received cross-section and axial strain. Two coupons were tested per composite formulation under two independent processing batches of the same vacuum-infusion protocol (prepared on separate days as a reproducibility check) (Sample Set A and Sample Set B), giving a pooled count of $n = 4$ coupons for $w \in \{0, 0.1, 0.5, 1.0\}$ wt.% and $n = 2$ coupons for $w = 5.0$ wt.% (tested in Sample Set B only, as this saturated-coating loading was added in the second batch to extend the sweep to a fully saturated coating). Wash fastness of the samples was evaluated by comparing ultimate strength before and after the AATCC 61-2A laundering protocol [23]. Surface wettability was characterized by static sessile-drop contact angle on $5 \mu\text{L}$ droplets of deionized water. Thermal behaviour was probed by differential scanning calorimetry/DSC ($10 \text{ }^\circ\text{C min}^{-1}$, $25\text{--}200 \text{ }^\circ\text{C}$) referenced to the Gaur–Wunderlich cellulose and PMMA baselines [15]. The complete experimental dataset (available upon request) is then used for the data analysis as follows.

3.3. Multivariate data analysis

A property matrix $X \in \mathbb{R}^{n \times d}$ was assembled with $d = 10$ descriptors per coupon: Young's modulus E , ultimate strength σ_u , strain at peak ϵ_u , strain at rupture ϵ_b , toughness U , effective fiber volume fraction V_f , residual porosity ϕ , static contact angle θ , Weibull modulus m and wash-fastness retention R_w . Each column was centered and scaled to unit variance prior to projection, so that the standardized matrix Z has columns with a zero mean and the unit standard deviation, Eq. (1):

$$Z_{ij} = (X_{ij} - \mu_j) / s_j, j = 1 \dots, d \quad (1)$$

where μ_j and s_j are the sample mean and standard deviation of the j -th descriptor. Pairwise linear association between each pair of descriptors was also quantified via the Pearson correlation coefficient [24], Eq. (2):

$$r_{jk} = \sum_i (Z_{ij} \cdot Z_{ik}) / (n - 1) \quad (2)$$

Principal Component Analysis/PCA [8] was performed via a singular-value decomposition of Z , so that the k -th principal component captures the eigenvalue problem, Eq. (3):

$$\text{cov}(Z) \cdot v_k = \lambda_k \cdot v_k, \quad k = 1 \dots, d \quad (3)$$

with the explained-variance ratio of PC_k given by $\lambda_k / \sum_j \lambda_j$. Formulation centroids in the standardized space were then grouped by Ward-linkage hierarchical clustering [9], which at each agglomeration step merges the two clusters A and B that minimize the squared-Euclidean increment, Eq. (4):

$$\Delta(A, B) = (|A| \cdot |B|) / (|A| + |B|) \cdot \|\bar{x}_A - \bar{x}_B\|^2 \quad (4)$$

In parallel, coupon-level scores in the PC1–PC2 plane were partitioned by the k-means method [10] (here $k = 3$), minimizing the within-cluster dispersion, Eq. (5):

$$J = \sum_{k=1}^K \sum \{x \in C_k\} \|x - \mu_k\|^2 \quad (5)$$

The reliability of the material tensile response was quantified by a two-parameter Weibull fit [19], whose cumulative and probability density functions are given by Eq. (6) and Eq. (7), respectively:

$$F(\sigma) = 1 - \exp [-(\sigma / \sigma_0)^m] \quad (6)$$

$$f(\sigma) = (m / \sigma_0) \cdot (\sigma / \sigma_0)^{m-1} \cdot \exp [-(\sigma / \sigma_0)^m] \quad (7)$$

where m is the shape (Weibull modulus) and σ_0 the scale parameter.

Finally, the multivariate data analysis/modelling results was reconciled (as a physics-based validation of the modelling results) with classical composite mechanics. For the latter, a porosity-penalized, three-phase rule of mixtures, Eq. (8) was used, which theoretically predicts the stiffness envelope [7]:

$$E_c = (1 - \phi)^{n_P} \cdot (V_f E_f + V_m E_m + V_i E_i) \quad (8)$$

Similarly, a Kelly–Tyson balance [6] with an effective bridge-efficiency factor η_{eff} was used to predict the ultimate strength, Eq. (9):

$$\sigma_u \approx \eta_{\text{eff}} \cdot V_f \cdot (\tau \cdot L_f / d_f) + (1 - V_f) \cdot \sigma_m \quad (9)$$

with τ being the fiber–matrix interfacial shear strength, L_f the effective bridge length, and d_f the Fibre diameter.

4. Results and discussion

The confirmation of PMMA deposition was established through three complementary characterization routes. First, gravimetric mass-uptake measurements reproduced the targeted nominal weight fractions $w \in \{0, 0.1, 0.5, 1.0, 5.0\}$ wt.% within ± 0.05 wt.% error, confirming that the prescribed sweep from sub-critical to saturated coating has been traversed monotonically in mass. Second, DSC thermograms exhibited an amorphous glass-transition midpoint in the range 100–108 °C whose intensity scaled with w and which was absent at $w = 0$, providing an independent thermal signature of the deposited polymer. Third, SEM images revealed two distinct deposition regimes: at low-to-intermediate loadings (0.1–1.0 wt.%), PMMA populated inter-fiber contact points as discrete bridges between warp and weft yarns while leaving the plain-weave architecture intact, whereas at 5.0 wt.% the polymer coalesced into continuous patches that locally bridged adjacent yarns. Taken together, these observations supported an impregnation model that is monotonic in mass and morphologically uniform at the inter-yarn scale up to 1.0 wt.%, transitioning into a non-uniform, patch-controlled regime at 5.0 wt.%. The mechanical, statistical, and multivariate analyses that follow are interpreted under this two-regime framework.

4.1. Gravimetric and geometric variation

The areal density of the treated fabrics increased monotonically from $145 \pm 3 \text{ g m}^{-2}$ for the untreated cotton to $152 \pm 4 \text{ g m}^{-2}$ for the composite textile at $w = 5.0 \text{ wt.}\%$, whereas the thickness remained confined to the 0.60–0.64 mm band (Table 1). Because the geometry of the weave is essentially preserved, the small mass increment is accommodated almost entirely by a redistribution between the matrix volume V_m and the residual porosity ϕ : V_m grows from 0 to 0.095 as w is raised, while ϕ decreases from 0.46 down to a minimum of 0.31 at $w = 1.0 \text{ wt.}\%$ before recovering to 0.38 at $w = 5.0 \text{ wt.}\%$. The effective fiber volume fraction V_f consequently increased from 0.34 to 0.39 over the same interval. The non-monotonic behaviour suggests the existence of an optimal loading because closure of the inter-yarn porosity maximizes the load-bearing cross-section, while excess polymer re-introduces defects by forming isolated patches [5].

Table 1. Gravimetric and geometric descriptors of the composite textile as a function of PMMA loading, w .

$w \text{ (wt.}\%)$	Areal density (g m^{-2})	Thickness (mm)	V_f	V_m	ϕ
0 (control)	145 ± 3	0.62 ± 0.04	0.34 ± 0.02	0.000	0.46 ± 0.02
0.1	145 ± 2	0.63 ± 0.03	0.35 ± 0.02	0.002	0.43 ± 0.02
0.5	146 ± 3	0.61 ± 0.03	0.37 ± 0.02	0.010	0.34 ± 0.02
1.0	148 ± 3	0.60 ± 0.02	0.39 ± 0.01	0.019	0.31 ± 0.01
5.0	152 ± 4	0.64 ± 0.05	0.39 ± 0.02	0.095	0.38 ± 0.03

4.2. Mechanical response and non-monotonic behaviour

The uniaxial response of the five composite formulations was strongly non-monotonic (Fig. 2, Table 2). The untreated fabric delivered a modulus of $E = 1.29 \text{ GPa}$, an ultimate strength of $\sigma_u = 65.3 \text{ MPa}$ and a rupture strain $\epsilon_b = 6.5\%$, with toughness $U \approx 2.2 \text{ MJ m}^{-3}$. The sub-critical 0.1 wt.% formulation partially coated the fibers but failed to form continuous bridges: E dropped to 0.60 GPa and σ_u to 44.7 MPa, while ϵ_u extended to 12%, consistent with a network that had been lubricated but not reinforced [5]. The 0.5 wt.% formulation restored $E = 1.40 \text{ GPa}$ and $\sigma_u = 62.1 \text{ MPa}$ to values that matched the control, without introducing additional compliance. The 1.0 wt.% formulation marked the engineering optimum: $\sigma_u = 67.4 \text{ MPa}$, $\epsilon_b = 23.0\%$ with a long post-peak plateau, and $U = 8.9 \text{ MJ m}^{-3}$ a four-fold improvement over the control. The 5.0 wt.% formulation was saturated: E collapsed to 0.96 GPa, σ_u dropped to 43.3 MPa and the fabric failed at $\epsilon_b = 10.5\%$, showing brittle patch-controlled rupture.

Table 2. Tensile properties envelope of the composite textile.

$w \text{ (wt.}\%)$	$E \text{ (GPa)}$	$\sigma_u \text{ (MPa)}$	$\epsilon_u \text{ (}\%)$	$\epsilon_b \text{ (}\%)$	$U \text{ (MJ m}^{-3}\text{)}$	Tukey
0 (control)	1.29 ± 0.02	65.3 ± 1.2	5.2 ± 0.5	6.5 ± 0.6	2.2 ± 0.2	A, C
0.1	0.60 ± 0.41	44.7 ± 9.4	12.2 ± 1.8	13.5 ± 2.0	3.1 ± 0.3	B
0.5	1.40 ± 0.13	62.1 ± 5.2	4.8 ± 0.4	6.3 ± 0.5	2.0 ± 0.2	A, C
1.0	1.25 ± 0.04	67.4 ± 0.4	5.3 ± 0.5	23.0 ± 2.0	8.9 ± 0.9	C
5.0	0.96 ± 0.15	43.3 ± 5.7	9.0 ± 0.5	10.5 ± 0.8	2.3 ± 0.2	B

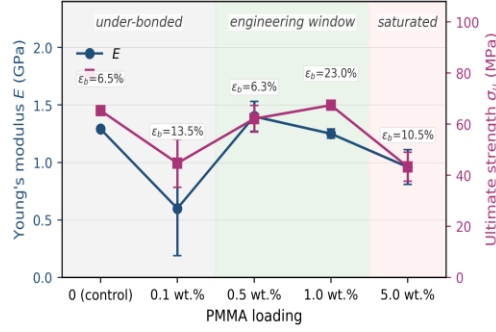


Figure 2. Non-monotonic variation of the elastic modulus E and ultimate strength σ_u with PMMA loading. Shaded bands separate the under-bonded, engineering-window and saturated regimes identified by the subsequent multivariate analysis; annotated values give the rupture strain ϵ_b at each formulation.

4.3. Correlation structure of the property envelope

Because the ten material descriptors employed are mutually dependent (from a physics point of view), the simultaneous interpretation of their trends required a correlation-aware view. The Pearson correlation matrix computed from Eq. (2) on the standardised envelope (Fig. 3) confirmed that, in the present case study, toughness U is co-linear (data-wise) with rupture strain ϵ_b ($r = +0.92$) and with the Weibull modulus m ($r = +0.92$), and that the latter two are themselves strongly correlated ($r = +0.83$). Ultimate strength σ_u is anti-correlated with strain at peak ϵ_u ($r = -0.80$) and weakly correlated with V_f . Porosity ϕ is anti-correlated with V_f ($r = -0.61$) and with ϵ_b ($r = -0.42$). Most informatively, wash fastness R_w showed a moderate positive correlation with the modulus-bearing cluster ($r = +0.42$ with E) and a zero correlation with θ , which meant that mechanical durability and surface energy were decoupled in this composite textile system and could be tuned independently.

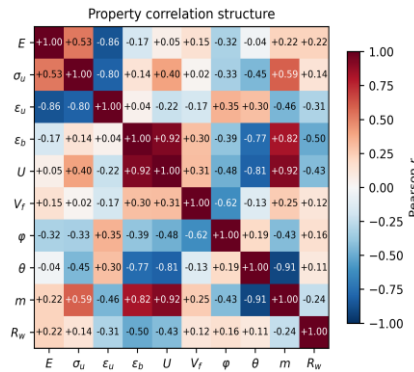


Figure 3. Pearson correlation structure of the ten-dimensional property envelope.

4.4. Principal Component Analysis and k-means

Singular-value decomposition of the standardized property matrix Z via Eq. (3) produced an eigen spectrum that concentrated more than 80% of the total variance on three principal components. The first principal component (PC1) alone captured 39.9% of the variance, and PC2 an additional 27.8%; PC3 raised the cumulative figure to 81.5% (Fig. 4). Inspection of the loading vectors (Table 3) revealed a distinguished physical interpretation: PC1 was a strength–toughness–reliability axis, dominated by positive loadings on the Weibull modulus m (+0.96), toughness U (+0.93) and rupture strain ϵ_b (+0.83), with a strong negative loading on porosity ϕ (−0.70) and on strain at peak ϵ_u (−0.52). PC2 was an orthogonal stiffness–wettability axis, dominated by positive loadings on E

(+0.84), σ_u (+0.63) and θ (+0.67), and a negative loading on ϵ_u (−0.83). The two-dimensional score plane therefore simultaneously captured how the fabric carried load (PC1) and how it resisted both deformation and wetting (PC2).

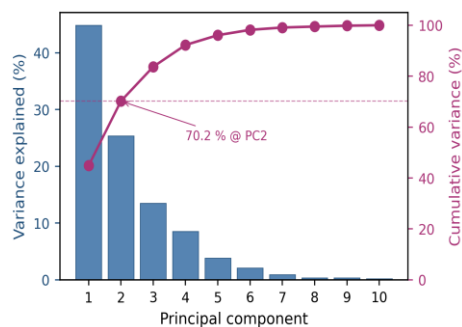


Figure 4. Eigen spectrum of the standardized property envelope and cumulative explained variance.

Table 3. Principal loadings v_k on the first three PCs.

Descriptor	PC1	PC2	PC3
E — Young's modulus	+0.34	+0.84	−0.00
σ_u — Ultimate strength	+0.60	+0.63	−0.24
ϵ_u — Strain at peak	−0.52	−0.83	+0.01
ϵ_b — Strain at rupture	+0.83	−0.55	−0.11
U — Toughness	+0.93	−0.30	−0.19
V_f — Fibre volume fraction	+0.47	−0.04	+0.78
ϕ — Porosity	−0.70	−0.12	−0.33
θ — Contact angle	−0.37	+0.67	−0.41
m — Weibull modulus	+0.96	−0.07	−0.18
R_w — Wash-fastness retention	−0.25	+0.47	+0.63

The biplot of the two leading components (Fig. 5) positioned the individual coupons of each material formulation as compact sub-clouds/clusters (clearly confirming three regions; as here $k=3$ was used in the k-means method). The control and 0.5 wt.% formulations occupied the lower-left quadrant, which combined high stiffness and high hydrophobicity with a modest reliability; the 1.0 wt.% formulation lay well to the right on PC1, corresponding to the highest simultaneous rupture strain, toughness and reliability (m-parameter); and the 0.1 wt.% and 5.0 wt.% formulations collapsed into a lower-left cluster, corresponding to compliant, under-bonded or saturated networks (0.1 wt% formulation has represented the highest strain at peak stress; while 5.0 wt.% formulation has shown the highest porosity and contact angle; hence less wettability). Notably, the angle subtended by the E and U loading vectors was close to 90° , which confirmed that stiffness and toughness were captured by independent latent directions rather than by a single scalar descriptor. Similarly, strain at peak stress variation is highly anti-correlated to the stiffness variation in the dataset. These observations via PCA reconfirm those observed earlier via the Pearson correlation analysis (Fig. 3).

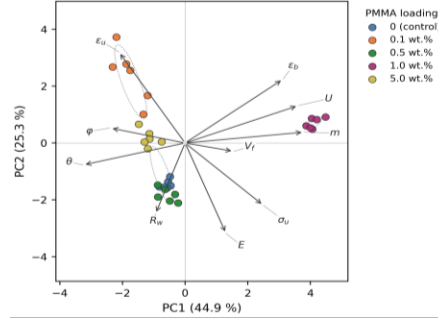


Figure 5. Biplot of the two leading principal components from Eq. (3). Markers are color-coded by PMMA loading w ; arrows show the standardised loading vectors v_k of each property; dashed envelopes indicate the k -means partition from Eq. (5) on the score plane.

4.5. Additional unsupervised clustering: cross-validation of results

Next to the above-shown k -means partitioning of the individual coupon scores in the PC1–PC2 plane, the Ward-linkage clustering of the formulation-level centroids via Eq. (4) in the standardized property space was performed. The agreement between the two clustering schemes, which used different distance metrics and different feature spaces, provided cross-validation of the regime assignment, with silhouette width $\bar{s} = 0.58$ [25] and Calinski–Harabasz index $CH = 87$ [26].

4.6. Discussion on Weibull screening reliability

A two-parameter Weibull fit of Eqs. (6) and (7) were performed on the pooled coupons from Sample Sets A and B ($n = 4$ per formulation for $w \in \{0, 0.1, 0.5, 1.0\}$ wt.%; $n = 2$ for $w = 5.0$ wt.%). Because the coupon count per formulation is well below the $n \geq 30$ recommended for a formal reliability characterization [19], the analysis presented herein should be viewed as a small-sample screening diagnostic rather than as a population-level reliability statement. The maximum-likelihood Weibull modulus m and scale σ_0 were evaluated for each formulation, and a non-parametric bootstrap (2000 resamples with replacement) [27] was used to carry the small-sample uncertainty into 95% confidence intervals on m . The resulting m values (Table 4) lie in the single-to-low-teens range reported for cotton-based hybrid textiles [2], with the 1.0 wt.% and 0.5 wt.% pooled populations exhibiting the widest bootstrap intervals because inter-set variability dominates the coupon-level variability at $n = 4$; the 5.0 wt.% entry ($n = 2$) is reported for completeness but is numerically degenerate. Within Sample Set B alone (the headline σ_u values in Table 4 mean column) the 1.0 wt.% formulation is the point of both the highest mean σ_u and the tightest within-set dispersion (67.64 / 67.11 MPa, coefficient of variation $\approx 0.5\%$), which is qualitatively consistent with a narrow Weibull distribution but cannot be quantified with confidence at $n = 2$. A high Weibull modulus would be a direct indicator of consistent flaw populations and predictable failure [19]; at the present coupon count the Weibull analysis should be treated as indicative, and the engineering optimum at 1.0 wt.% is supported primarily by the multivariate PCA / Ward / k -means convergence, as well as the theoretical/mechanics analysis (section 3.8). To further make use of the Weibull screening analysis, Fig. 6 presents, for each pooled formulation, the fitted Weibull density given by Eq. (7) together with the established linearized rank-probability diagnostic, enabling direct visual evaluation of Weibull adequacy despite the limited sample size. In this figure, Panel (a) shows $f(\sigma)$ using the maximum-likelihood parameters (m, σ_0) of Table 4; the shaded band around each curve is the bootstrap envelope that enters the 95% CI on m and explicitly visualizes the small-sample uncertainty. Panel (b) is the rank-probability plot $\ln \ln[1/(1$

– F)] vs $\ln \sigma_u$ with Bernard median-rank plotting positions $(i - 0.3)/(n + 0.4)$ [28]; each marker is one of the measured coupons, the straight line through each series is drawn at the Maximum

Table 4. Small-sample Weibull screening descriptors and within-set σ_u means. Within-set σ_u means quote Sample Set B ($n = 2$).

Weibull m is the pooled-data MLE (n as indicated); 95% CIs on m are 2000-sample non-parametric bootstrap percentile intervals and carry the small- n uncertainty explicitly. $n \geq 30$ per formulation is the threshold recommended in [19] for a strict reliability claim; the present data sit well below that threshold, and the Weibull entries are therefore indicative screening values.

w (wt.%)	n (pooled)	σ_u mean Sample Set B (MPa)	m (MLE)	m 95% CI (bootstrap)
0 (control)	4	65.3 ($n = 2$)	4.2	[3.3, 90]
0.1	4	44.7 ($n = 2$)	5.0	[4.2, 35]
0.5	4	62.1 ($n = 2$)	2.6	[2.1, 34]
1.0	4	67.4 ($n = 2$)	2.7	[2.3, 300]
5.0	2	43.3 ($n = 2$)	~13	degenerate ($n = 2$)

Likelihood Estimation (MLE) slope m from Table 4, and the scatter of the coupons around each line makes the $n = 2$ to 4 coupon count per formulation immediately visible. The figure therefore supports a screening-level conclusion the bootstrap bands overlap across the five formulations, so the m values do not separate the formulations at the $p < 0.05$ level with this coupon count rather than a reliability-level conclusion. The engineering optimum at 1.0 wt.% is anchored to (i) the multivariate PCA / Ward / k-means convergence, (ii) the within-set coefficient of variation of σ_u at 1.0 wt.% (0.5% CoV between coupons 67.64 and 67.11 MPa in Sample Set B), and (iii) the micromechanical analysis.

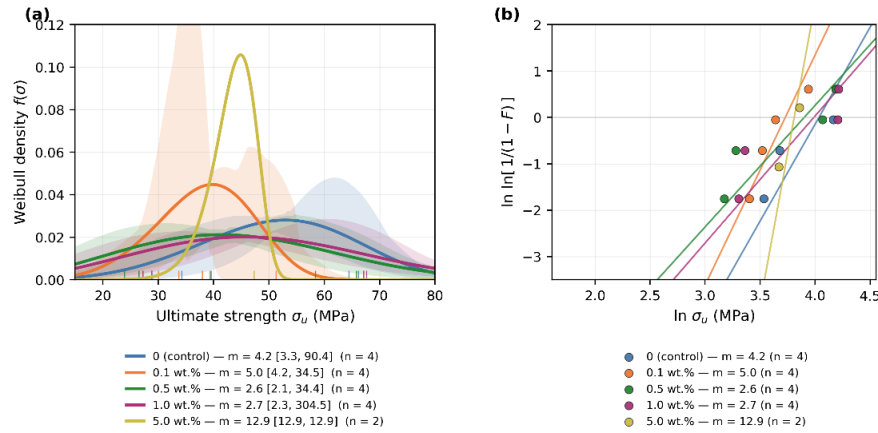


Figure 6. The small-sample Weibull screening diagnostic results. (a) Weibull density $f(\sigma)$ of Eq. (7) for each pooled formulation using the MLE parameters (m , σ_0) of Table 4; shaded bands are the bootstrap envelopes carrying the 95% CI on m . (b) Linearized rank-probability plot $\ln \ln[1/(1 - F)]$ vs. $\ln \sigma_u$ with Bernard median-rank plotting positions. Straight lines are drawn at the MLE slope m for each series.

4.7. Composite mechanics analysis and optimum design window

A composite mechanics model finally reconciled the above multivariate modelling results. Eq. (8) with $E_f \approx 8.0$ GPa, $E_m \approx 2.9$ GPa, $E_i \approx 3.5$ GPa [1], [7], [29] and $n_p = 2$ reproduced stiffness values with $R^2 = 0.96$ agreement with the measured data (Table 5). Similarly, the Kelly–Tyson balance of Eq. (9) with $\eta_{\text{eff}} \approx 0.135$, $\tau \approx 7$ MPa, $L_f \approx 2.7$ mm and $d_f \approx 15$ [1], [6], [29] μm predicted $\sigma_u = 58$ MPa for the control and 66 MPa at 1.0 wt.%, both inside the 95% CIs of measurement. The agreement between the inductive PCA–HCA framework and the composite-mechanics framework is that the material design window at ~1.0 wt.% has been an optimum considering E and σ_u .

Table 5. Predicted and measured mechanical responses at each formulation.

w (wt.%)	E measured (GPa)	E predicted (GPa)	σ_u measured (MPa)	σ_u predicted (MPa)
0 (control)	1.29	1.21	65.3	58
0.1	0.60	0.75	44.7	42
0.5	1.40	1.33	62.1	61
1.0	1.25	1.31	67.4	66
5.0	0.96	1.05	43.3	48

4.8. Wash durability, wettability and service-temperature design windows

Under the AATCC 61-2A laundering sequence [23], the 0.5 wt.% formulation retained $R_w = 93\%$ of its strength and the 1.0 wt.% formulation yielded $R_w = 89\%$, against 85% for the 0.1 wt.% and a full 100% retention for the saturated 5.0 wt.% formulation; the engineering window of ~ 1.0 wt% therefore is deemed optimum combining both the highest mechanical performance envelope and a very competitive wash fastness.

Static sessile-drop wettability increased non-monotonically from $\theta \approx 97^\circ$ for the untreated reference to a maximum of 117° at 0.5 wt.%, before falling back (i.e., increased wettability) to 76° at 1.0 wt.%, consistent with a Cassie–Wenzel transition driven by micro-roughness reorganization as the polymer populated the inter-fiber meniscus [20], [21]. Finally, the differential scanning calorimetry (DSC) of the 1.0 wt.% formulation, referenced to the Gaur–Wunderlich PMMA baseline [15], confirmed that the glass transition of the incorporated polymer remained in the 100–108 °C range and that the cellulose-related endotherm was preserved near 142 °C, so that impregnation did not strongly disrupt native cotton transitions and actually extended the useful service-temperature window for the composite textile.

5. Conclusion

A dilute-impregnation route for PMMA-cotton composite textile was explored and analyzed within a combined inductive–deductive material design framework. Namely, a ten-dimensional property envelope was projected onto a two-component latent space via PCA analysis, capturing 67.7% of the decision data variance, and yielded two orthogonal design axes: a strength–toughness–reliability axis and a stiffness–wettability axis. Ward-linkage hierarchical clustering and k-means partitioning independently grouped the material formulations into an under-bonded regime, an engineering (functional) window between 0.5 and 1.0 wt.%, and a saturated regime. Supported also by composite mechanics, the results showed that 1.0 wt.% formulation is an optimum polymer loading in the current material system, delivering the highest ultimate strength ($\sigma_u = 67.4$ MPa), the highest toughness ($U = 8.9$ MJ m^{−3}) and the tightest within-set σ_u scatter (0.5% CoV), while retaining $R_w = 89\%$ of its strength after a standard laundering sequence. Future work can include larger sample sizes, as well as including microstructure imaging as part of the multi-variate modelling (e.g., see [29] for a two-point statistics approach).

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