Incorporating Crack-Parallel Stress Effects into a Spectral Stiffness Microplane Fracture Model

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1. Abstract

Recent experimental studies have demonstrated that Crack-Parallel Stress (CPS) can significantly influence the fracture response of quasibrittle materials such as concrete [1], [2], [3] and Carbon-Fiber-Reinforced Polymer (CFRP) [4], [5]. In symmetric cross-ply CFRP laminates, the recently proposed Gap Test revealed reductions in apparent fracture energy of up to 37% when substantial crack-parallel compression was introduced during crack growth [4], [5]. However, as demonstrated in the present work, the influence of CPS on damage propagation in composite structures is not sufficiently captured by conventional homogenized constitutive models, which are required to model full-scale structures without prohibitive computational cost.

This work presents an extension of the Spectral Stiffness Microplane (SSM) constitutive framework [6] intended to incorporate the influence of CPS while maintaining computational efficiency suitable for structural-scale finite element simulations. The proposed formulation introduces a phenomenological coupling between fracture evolution and the local CPS state through a modified traction-separation law. Numerical simulations are performed to evaluate the ability of the model to reproduce experimentally observed trends in fracture energy reduction.

The results demonstrate that the proposed approach captures the primary influence of CPS on crack growth. The expanded modeling capability revealed that elastic unloading within the polypropylene pads used in the Gap Test can contribute significantly to the measured structural response, an effect that is important for practitioners interpreting apparent fracture energy measurements. The study highlights the importance of incorporating multidirectional stress effects into homogenized constitutive descriptions for the failure of composite structures.

2. Introduction

CFRP composites are increasingly employed in aerospace, marine, civil, and energy applications because of their high specific stiffness, strength, and corrosion resistance [7]. As the characteristic dimensions of these structures continue to increase, the computational cost associated with high-fidelity mesoscale simulations becomes prohibitive. For many structural analyses, fully homogenized constitutive approaches remain the only practical option for predicting structural response at large scales.

However, homogenized formulations often neglect the influence of multiaxial stress states on fracture evolution. Recent investigations using the Gap Test demonstrated that crack-parallel compression can substantially reduce the apparent fracture resistance of symmetric cross-ply laminates [4], [5]. The experiments showed that increasing CPS can accelerate fracture propagation even when the dominant loading condition is nominally mode I.

Conventional constitutive models underestimate the sensitivity of CFRP fracture processes to multidirectional loading effects. For example, Fig. 1 shows that the SSM model [6] includes a weak effect of multiaxial stress, while the Hashin damage model [8], [9] does not include any effect. In the case of the SSM model, the effect is explained by the multiaxial failure envelope included in the model [6]. The failure envelope allows for accurate prediction of material strength but is not sufficient to represent the full effects of CPS on crack propagation in laminated structures.

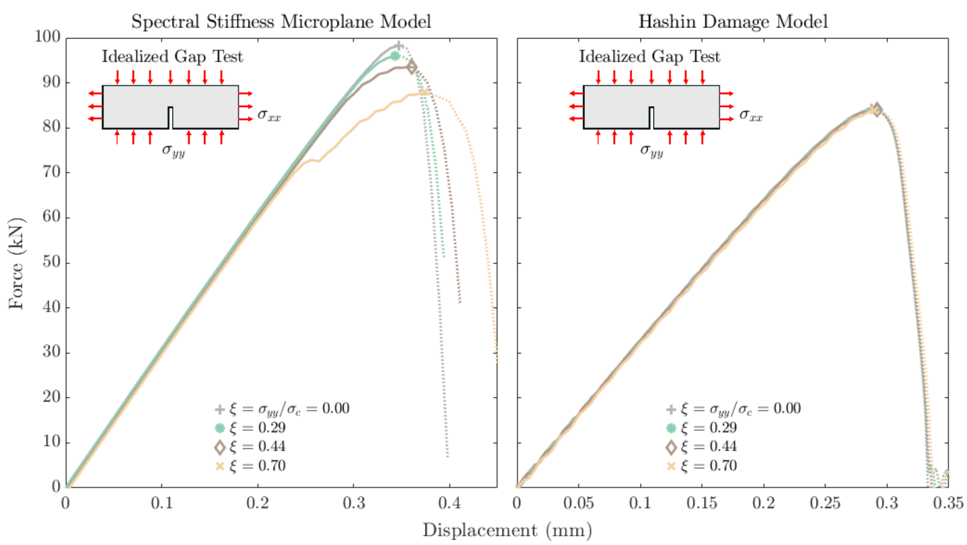

Figure 1. The spectral stiffness microplane model (left) predicts a small decrease in structural strength in the presence of a crack-parallel compression, while the Hashin damage model (right) does not. The level of applied compression is described by ξ: the externally applied crack-parallel stress normalized by the compressive strength of the specimen.

Motivated by these findings, the present work extends the SSM constitutive formulation to incorporate CPS effects during fracture evolution. The objective is to develop a phenomenological enhancement capable of reproducing experimentally observed trends while preserving the computational efficiency required for large structural simulations. The work focuses on maintaining compatibility with homogenized finite element workflows and avoiding the substantial computational expense associated with explicitly resolved mesoscale fracture modeling.

3. Theoretical Framework

The constitutive formulation employed in this work is based on the SSM framework previously developed for anisotropic quasibrittle materials [6]. Within this approach, constitutive relations are defined on multiple microplanes distributed over the unit sphere, enabling the representation of orientation-dependent damage processes while maintaining robust numerical behavior. The spectral decomposition of the elastic tensor permits efficient treatment of strongly anisotropic material behavior and facilitates the introduction of physically motivated inelastic mechanisms. Details of the formulation are given in Salviato et. al. [6].

In the original constitutive framework, fracture evolution is governed through a traction-separation relation that is used in combination with the crack band model [10], [11], [12], [13], [14], [15], [16] such that fracture is modeled with mesh objectivity. The primary enhancement introduced in the present work is a modification to the traction-separation law intended to account for the influence of CPS. Experimental observations from the Gap Test demonstrated that increasing crack-parallel compression systematically reduces the apparent fracture energy measured during crack propagation [4], [5]. This trend suggests that the Fracture Process Zone (FPZ) is sensitive to multiaxial stress interactions that are not represented in conventional homogenized constitutive descriptions. Accordingly, the updated traction-separation law is made to be dependent on the local CPS state.

The relevant constitutive law assumes the effective stress σ to be incrementally linearly elastic with respect to the effective strain ε through the effective modulus λ, and a strain-dependent stress limiting boundary given by:

$$\sigma_b = s\,\exp\left[-\left(\frac{\langle \varepsilon_{\max} - \varepsilon_0 \rangle}{k}\right)^a\right]$$

is used to impose an exponential softening behavior once the elastic limit in tension ε_0 is exceeded. In the above, s is the peak stress value, $\varepsilon_{\max}$ is the maximum effective strain, k and a are parameters that determine the shape of the exponential softening curve, and $\langle x \rangle = \max(x, 0)$ is defined in the Macaulay sense. The associated fracture energy G_f can be calculated according to:

$$\frac{G_f}{h} = \frac{1}{2}\varepsilon_0 s + \int_{\varepsilon_0}^{\infty} \sigma_b \; d\varepsilon = \frac{1}{2}\varepsilon_0 \mathrm{s} + \frac{\mathrm{ks}}{\mathrm{a}}\Gamma\left(\frac{1}{\mathrm{a}}\right)$$

where h is the finite element characteristic length and Γ is the gamma function. Thus, the fracture energy is found to be a linear function of the softening parameter k. Representative curves for the constitutive law are given in Fig. 2 for various values of the parameter k.

The idea is to modify the constitutive law so that the parameter k, and therefore the amount of energy dissipated by the finite element during fracture, depends on the local CPS state within the element. To achieve this, the local strain state is measured at the microplane level according to the quantity defined by:

$$R = \left|\frac{\varepsilon_N^{(1)} g(\theta, \varphi)}{\varepsilon_N^{(2)} f(\theta, \varphi)}\right|$$

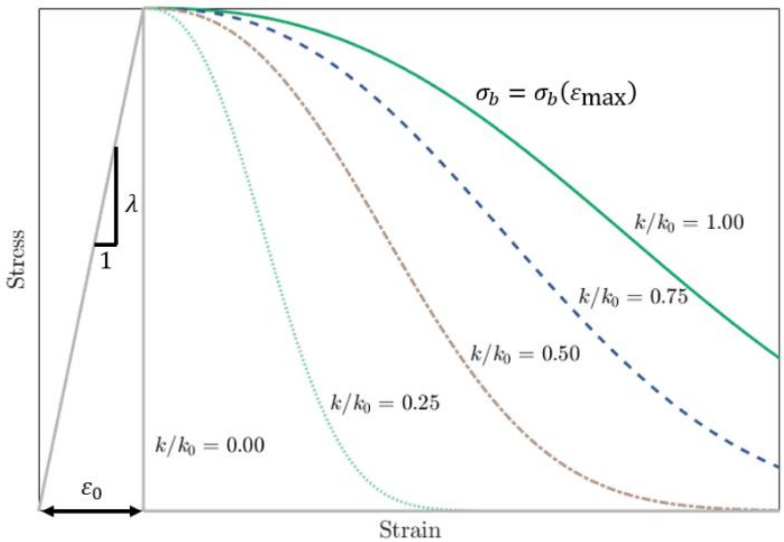

Figure 2. A diagram representing the exponential softening law used in the spectral stiffness microplane model and its dependence on parameter k.

where $\varepsilon_N^{(I)}$ is the normal component of the I^{th} microplane eigenstrain as detailed in Salviato et. al. [6] and f and g are functions of the microplane orientation defined by angles θ and φ that simply serve as normalization factors. As demonstrated by Salviato et. al. [6], the mode 1 eigenstrain is excited by volumetric-like deformations, while the mode 2 eigenstrain is excited by deviatoric-like deformations. Therefore, the ratio of the two may be used to quantify the local strain state within an element. In fact, it can be shown that, for a given material, the quantity R takes on a constant value η in a state of uniaxial strain, while $R \approx 0$ in a state of high deviatoric strain and $R \gg \eta$ in a highly volumetric strain state. Therefore, the measure R allows for discrimination between biaxial strain states intrinsic to mode I crack opening and those that are altered by CPS.

The proposed formulation preserves the general structure of the original constitutive model while introducing a phenomenological coupling between crack opening and crack-parallel loading. Under increasing crack-parallel compression, the model accelerates the rate of softening in the element response and therefore reduces the energy dissipated during fracture evolution, thereby reproducing the experimentally observed decrease in apparent fracture resistance. Figure 3 demonstrates the difference in capability between the original microplane model and the extended model.

4. Methods

Numerical simulations were performed using the Multiphysics Analyses of the Response of Structures solver [17], [18], a finite element framework supporting advanced constitutive formulations and parallel computation. The analyses modeled single-edge-notched Toray T800H/3900-2 cross-ply laminates subjected to both conventional four-point bending and Gap Test loading conditions corresponding to the experimental configuration introduced by Brockmann and Salviato [4], [5]. A total of nine simulations were conducted. Three geometrically scaled laminate sizes were analyzed, each subjected to three different levels of crack-parallel compression.

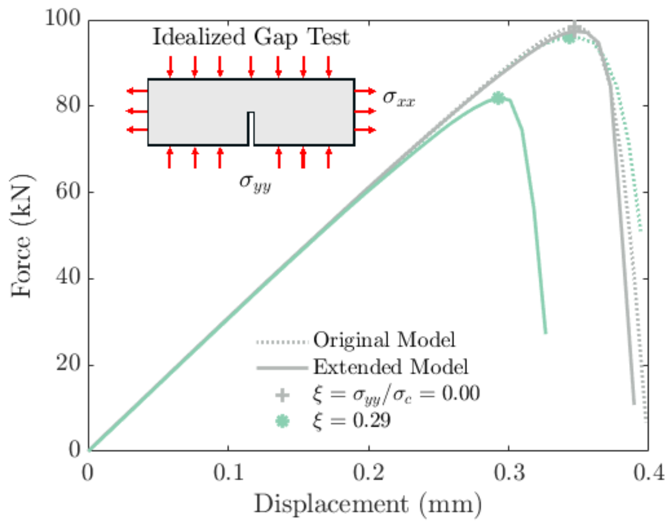

Figure 3. The extended spectral stiffness microplane model predicts a much stronger effect of crack-parallel compression on the structural strength of the notched specimen in the idealized Gap Test.

The models employed a fully three-dimensional representation (Fig. 4) using structured meshes of eight-node hexahedral solid elements. A half-symmetry boundary condition about the laminate midplane was used to reduce computational cost, and moderate mass scaling was introduced while maintaining negligible kinetic energy relative to internal energy throughout the simulations. Post-processing and visualization were conducted using ParaView [19], [20] and MATLAB [21].

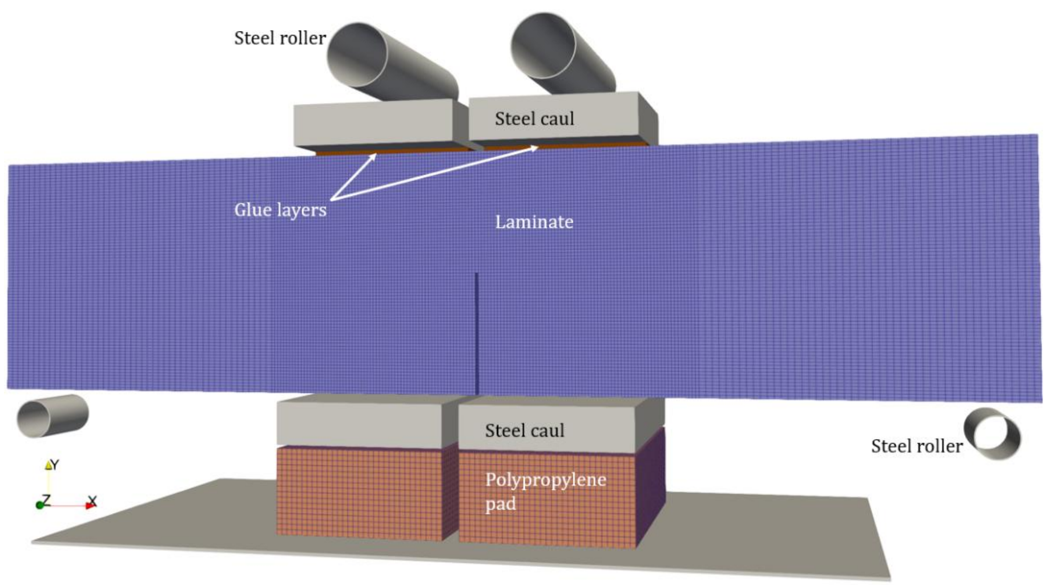

Figure 4. The numerical mesh for the medium size geometry.

The laminate constitutive behavior was modeled using the extended SSM formulation described previously. Where possible, the elastic and strength properties were derived from lamina-level material data using classical lamination theory [7], while the fracture energy was calibrated to reproduce the experimentally observed value under nominal conditions, i.e., under zero CPS.

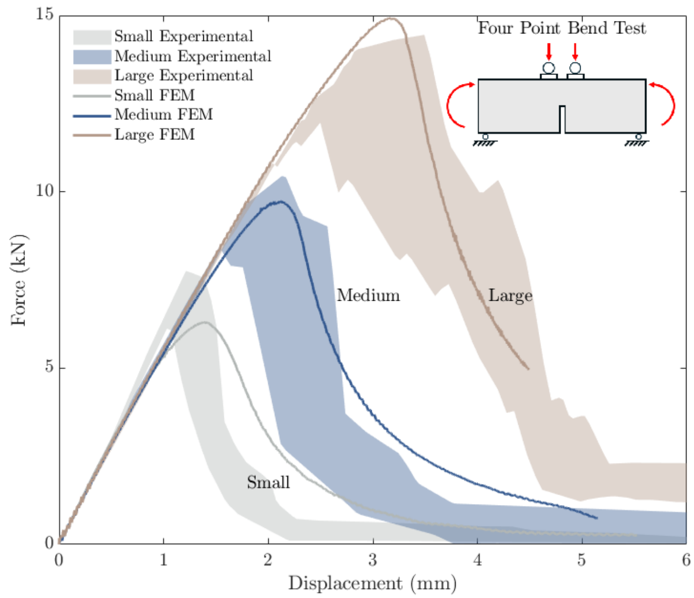

Figure 5. Results of the conventional four-point bend simulations using the extended spectral stiffness microplane model. Finite element results (solid curves) are plotted together with the experimental range of data (shaded regions).

The crack band formulation embedded within the SSM model ensured mesh-objective fracture predictions. To accurately capture fracture evolution, the mesh was refined near the notch region where nonlinear behavior and damage localization were expected.

The polypropylene pads used to introduce crack-parallel compression were modeled using a J2 elastic-plastic constitutive law [22], [23] with experimentally informed hardening behavior. The adhesive layers between the laminate and steel cauls were modeled explicitly using elastic-plastic solid elements in order to distribute stresses and reduce artificial stress concentrations near loading interfaces. Steel rollers, cauls, and support surfaces were modeled as rigid bodies.

Contact interactions between rollers, cauls, pads, and laminate components were enforced using a penalty-based hysteresis contact formulation. Frictionless contact was assumed at the rollers, while friction coefficients were applied between the polypropylene pads and surrounding components to represent experimentally observed behavior. Perfect bonding constraints were imposed where adhesive attachment was intended.

5. Results

The numerical model successfully reproduced the experimental response for the conventional four-point bend configuration corresponding to zero crack-parallel compression. Figure 5 compares the simulated load-displacement curves for the small, medium, and large specimen geometries against the experimentally reported ranges.

The simulated structural stiffnesses agreed well with the experimental measurements. The numerical response exhibited slightly greater pre-peak nonlinearity than observed experimentally, which is consistent with the underlying microplane formulation. Because constitutive softening evolves independently on multiple microplanes depending on orientation, distributed micro-damage develops gradually prior to peak load. This feature allows the SSM framework to represent the formation and evolution of the FPZ with greater physical realism than conventional continuum damage formulations. Overall, the agreement between simulations and experiments provided confidence in the constitutive implementation, mesh design, loading rate selection, and contact formulations employed throughout the study.

The simulations involving crack-parallel compression introduced additional mechanical behavior associated with deformation of the polypropylene pads beneath the laminate. Because the measured reaction force in the Gap Test contains contributions from both pad compression and laminate bending, interpretation of the structural response requires care. The total reaction force may be written as $F_{\text{total}} = F_{\text{pads}} + F_{\text{rollers}}$ where F_{pads} arises from compression of the polypropylene pads and F_{rollers} corresponds to bending resistance of the laminate itself. Since the primary objective of the study is assessment of fracture behavior, subsequent analyses focus on the laminate bending contribution F_{rollers}, which permits direct comparison with the conventional four-point bend configuration.

Figure 6 summarizes the bending load responses for all specimen sizes at the two nonzero compression levels ($\xi = 0.29$ and $\xi = 0.44$). The simulations reproduced the experimentally observed reduction in peak load with increasing crack-parallel compression. All predicted peak loads fell within two standard deviations of the reported experimental means, while most were within a single standard deviation. The results demonstrate that the extended constitutive formulation captures the primary experimental trend: increasing crack-parallel compression reduces the apparent fracture resistance of the laminate.

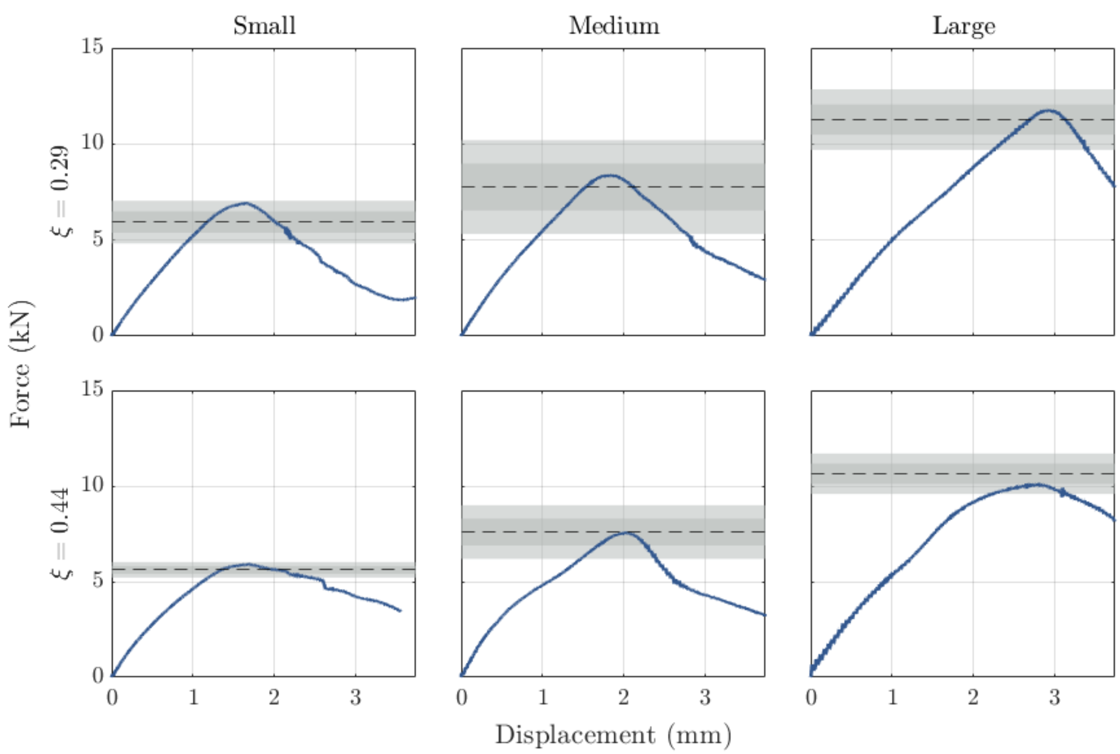

Figure 6. Results of the simulated Gap Tests for all three specimen sizes and both nonzero crack-parallel compression levels. Finite element data is plotted as the solid curves, while the experimental means and variance are indicated by the horizontal shaded bands.

6. Discussion

Overall, the extended SSM model showed strong agreement with the Gap Test experiments for cross-ply CFRP laminates. The modified traction-separation formulation was both necessary and sufficient to reproduce the experimentally observed sensitivity of apparent fracture resistance to crack-parallel compression. The proposed enhancement therefore provides a practical starting point for incorporating multiaxial fracture effects into homogenized constitutive descriptions suitable for structural-scale simulations.

An important observation from the simulations is that, despite the externally applied crack-parallel compression, the FPZ itself continued to develop under local crack-parallel tension. The role of the externally applied compression was not to reverse the local crack-tip stress, but rather to reduce the magnitude of the intrinsic tensile CPS generated during bending-induced fracture. Consequently, the effective fracture response depended on the combined influence of intrinsic stresses associated with crack opening and extrinsic stresses introduced by the experimental configuration. This distinction is important for interpretation of Gap Test experiments and, more broadly, for simulation of composite fracture under multiaxial loading conditions. The local stress state governing fracture evolution cannot be inferred solely from the externally applied loading configuration. Instead, the interaction between structural mechanics, material anisotropy, and crack-tip stress redistribution must be considered together.

The results also suggest that interpretation of Gap Test experiments depends strongly on the material system under investigation. In materials such as concrete, where the tensile strength is small relative to the compressive strength, externally applied crack-parallel compression can more readily dominate the local crack-tip stress state. In contrast, the cross-ply CFRP laminates considered here possess comparatively high tensile strength, making it more difficult for the externally applied compression to overcome the intrinsic tensile stresses generated during bending.

Importantly, the finite element analyses also revealed that partial elastic unloading of the polypropylene pads can occur during crack propagation. The release of strain energy stored within the pads contributes additional energy to the structural system and therefore promotes earlier crack propagation. This is seen if the fracture energy balance for quasi-static crack growth is considered:

$$\delta W^F = \delta W^{\text{ext}} - \delta U - \delta W^P,$$

where δW^F is the energy available to drive the fracture process, δW^{ext} is the external work supplied to the system, δU is the elastic strain energy within the system, and δW^P is the energy dissipated from the system via plastic deformation in the polypropylene pads. Consequently, the apparent reduction in fracture energy inferred from macroscopic experimental measurements may substantially overestimate the true reduction associated with the material fracture processes themselves.

The present simulations suggest that a significant portion of the experimentally inferred fracture energy reduction may be attributable to strain energy released from the polypropylene pads during bending-induced specimen rotation. These findings imply that the true reduction in intrinsic fracture resistance may be considerably smaller than would be inferred directly from the measured peak structural loads alone.

Nevertheless, the simulations still support the conclusion that CPS influences fracture evolution in cross-ply CFRP laminates, but further study of the phenomenon at higher levels of parallel stress are necessary to fully understand the effect on apparent fracture energy. Ongoing experimental efforts at the University of Washington using alternative loading configurations capable of applying substantially larger CPS suggest that the most pronounced reductions in fracture resistance occur once the applied CPS exceeds approximately 50%-60% of the relevant material strength.

7. Conclusions

This work presented an extension to the spectral stiffness microplane model that enables crack-parallel stress effects to be incorporated within a fully homogenized continuum formulation suitable for large-scale structural simulations. The enhanced constitutive framework was applied to cross-ply CFRP laminates subjected to Gap Test loading conditions and demonstrated good agreement with experimental observations.

Based on the analyses presented in this work, the following conclusions may be drawn:

- Existing fully homogenized continuum approaches are limited in their ability to capture the effects of crack-parallel stresses on fracture propagation in composite materials.
- The extended SSM model, in which the exponential softening law depends on the local biaxial strain state, successfully reproduced the experimentally observed influence of crack-parallel compression on apparent fracture resistance in CFRP laminates.
- Although crack-parallel compression was externally applied during the Gap Test, the fracture process zone developed under local crack-parallel tension. The applied compression primarily reduced the magnitude of the intrinsic tensile crack-parallel stress generated during bending.
- For the CFRP laminates considered here, compression levels greater than those investigated experimentally may be required to generate substantial crack-parallel compression directly within the fracture process zone.
- Elastic unloading of the polypropylene pads releases additional strain energy into the structural system, causing earlier crack propagation and exaggerating the apparent reduction in fracture energy inferred from experimental measurements.

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